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Effects of Variable Viscosity and Thermal Conductivity on Steady MHD Slip Flow of Micropolar Fluid over a Vertical Plate

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Abstract: Effects of variable viscosity and thermal conductivity on magnetohydrodynamic free convection slip flow and heat transfer of micropolar fluid over a vertical plate with viscous dissipation have been studied. The fluid viscosity and thermal conductivity are assumed to be vary as an inverse linear functions of temperature. The governing partial differential equations of motion are transformed into a system of ordinary differential equations using similarity transformations which are solved numerically for prescribed boundary conditions by shooting method. Numerical results for the velocity, angular velocity and temperature profile are shown graphically for various values of the parameters which gives the flow and heat transfer characteristics of the fluid. The results show that the variable viscosity and thermal conductivity have significant influence on the flow and heat transfer of the fluid.

Keywords: variable viscosity; thermal conductivity; slip flow; micropolar fluid; shooting method.

1. INTRODUCTION

Micropolar fluids are the fluids which contain micro-constituents, belonging to a class of fluids with non-symmetrical stress tensor called polar fluids. So these fluids can be defined as a viscous, non- Newtonian fluid, whose fluid elements exhibit micro-rotation. Eringen [1] first introduced the theory of micropolar fluid in 1964.

The study of micropolar fluid is very important as it has wide field of engineering applications such as oil exploration, geothermal extractions, polymer processing, micro-fluidics and many others. In particular, the study of slip flow has become field of active research due to its practical importance. The no-slip boundary condition may not be suitable for hydrophilic flows over hydrophobic boundaries. Also in mechanical engineering, partial slip can occur in channel with a coated or polished artificial heart values. This phenomenon is also common in the flow of blood.

An appreciable number of studies have been carried out on these flows under different flow conditions. Chaudhary and Jha [2] studied the effects of chemical reactions on MHD micropolar fluid flow past a vertical plate in slip flow regime. Das [3] analysed the slip effects on MHD mixed convection stagnation point flow of a micropolar fluid towards a shrinking vertical sheet. Das [4] also studied the slip effects on heat and mass transfer in MHD micropolar fluid flow over an inclined plate with thermal radiation and chemical reaction. An investigation on free convection flow of heat generating fluid through a porous vertical channel with velocity slip and temperature jump was carried out by Adesanya [5]. A study on the effects of slip and heat generation/absorption on MHD mixed convection flow of a micropolar fluid over a heated stretching surface was done by Mahmoud and Waheed [6]. Narayana and Ganadhar [7] discussed the problem of second order slip flow of a MHD micropolar fluid over an unsteady stretching surface. Mahmoud and Waheed [8] examined the MHD flow and heat transfer of a micropolar fluid over a stretching surface with heat generation (absorption) and slip velocity. Unsteady flow of radiating and chemically reacting MHD micropolar fluid in slip-flow regime with heat generation was studied by Abo-Dahab and Mohamad [9]. An analysis of a mathematical model on magnetohydrodynamic slip-flow and heat transfer over a non-linear stretching sheet was carried out by Das [10]. Zaib and Shafic [11] studied the slip effects on unsteady MHD stagnation point flow of a micropolar fluid towards a shrinking sheet with thermophoresis effect. Mukhopadhyay and Mandal [12] investigated the magnetohydrodynamic (MHD) mixed convection slip flow and heat transfer over a vertical porous plate.

In this study an attempt has been made to investigate the effect of variable viscosity and thermal conductivity on magnetohydrodynamic free convection slip flow and heat transfer of micropolar fluid over a vertical plate with viscous dissipation. The fluid viscosity and thermal conductivity are assumed to be vary as inverse linear functions of temperature. The governing partial differential equations of motion are transformed into a system of ordinary differential equations using similarity transformations which are solved numerically for prescribed boundary conditions by shooting method. Numerical results for the velocity, angular velocity and temperature profile are shown graphically for various values of the parameters.

2. MATHEMATICAL FORMULATION

We consider a steady free convection two dimensional viscous incompressible micropolar fluid over a vertical plate of very small thickness and much larger breadth. Let u and v be the component of velocity in x and y directions respectively where x- axis is considered along the plate and y-axis is taken normal to the x-axis as shown in the figure 1. A transverse uniform magnetic field B_0 is applied on the plate. The fluid properties are assumed to be constant, except for the fluid viscosity and thermal conductivity which are assumed to be inverse linear functions of temperature. Let, N be the micro-rotation component.

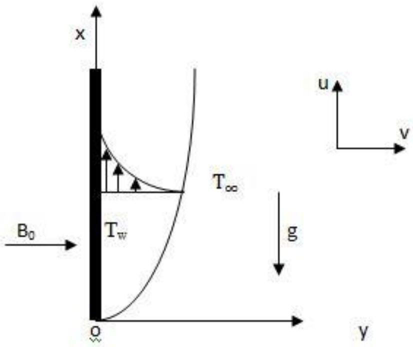

Figure 1: Flow configuration

Under the boundary layer assumptions, the governing equations are given below:

The equation of continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

The equation of motion:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\left[\mu\frac{\partial^2 u}{\partial y^2}+\frac{\partial \mu}{\partial y}\frac{\partial u}{\partial y}\right]+\frac{\kappa}{\rho}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial N}{\partial y}\right)$$
$$-\frac{\sigma B_0^2}{\rho}\left(u-u_\infty\right)+g\beta(T-T_\infty) \qquad (2)$$

The equation of angular momentum:

$$\rho j\left(u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}\right)=-\kappa\left(2N+\frac{\partial u}{\partial y}\right)+\gamma\frac{\partial^2 N}{\partial y^2} \qquad (3)$$

The energy equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho C_p}\left[\frac{\partial \lambda}{\partial y}\frac{\partial T}{\partial y}+\lambda\frac{\partial^2 T}{\partial y^2}+(\mu+\kappa)\left(\frac{\partial u}{\partial y}\right)^2\right] \qquad (4)$$

The boundary conditions for the problem are:

$$\left.\begin{array}{l} u=L_1\frac{\partial u}{\partial y}, v=0, T=T_w+D_1\frac{\partial T}{\partial y}, N=-\frac{1}{2}\frac{\partial u}{\partial y} \text{ at } y=0 \\ u\to u_\infty, T\to T_\infty, N\to 0 \text{ as } y\to\infty \end{array}\right\} \qquad (5)$$

where ρ is the fluid density, μ is the coefficient of dynamic viscosity, T is the fluid temperature, λ is the thermal conductivity, j is the micro-inertia per unit mass, C_p is the specific heat at constant pressure, σ is the electrical conductivity, γ is the spin gradient viscosity, κ is the kinematic micro-rotation viscosity, g is the acceleration due to gravity, β is the coefficient of thermal expansion, T_w is the temperature of the plate and T_∞ is the free stream temperature, $L_1=L(Re_x)^{\frac{1}{2}}$ is the velocity slip factor and $D_1=D(Re_x)^{\frac{1}{2}}$ is the thermal slip factor with L and D being the initial values of velocity and thermal slip factors having the same dimension of length, $Re_x=\frac{u_\infty x}{v_\infty}$ is the local Reynolds number.

Following, Lai and Kulacki [13] the fluid viscosity is assumed as,

$$\left.\begin{array}{l} \frac{1}{\mu}=\frac{1}{\mu_\infty}[1+\delta(T-T_\infty)] \\ \text{or,} \quad \frac{1}{\mu}=a\,(T-T_r) \\ \text{where } a=\frac{\delta}{\mu_\infty} \text{ and } T_r=T_\infty-\frac{1}{\delta} \end{array}\right\} \qquad (6)$$

where, μ_∞ is the viscosity at infinity, a and T_∞ are constants and their values depend on the reference state and thermal property of the fluid. T_r is transformed reference temperature related to viscosity parameter, δ is a constant based on thermal property of the fluid and $a<0$ for gas, $a>0$ for liquid.

Similarly, the thermal conductivity is considered as,

$$\left.\begin{array}{l} \frac{1}{\lambda}=\frac{1}{\lambda_\infty}[1+\xi(T-T_\infty)] \\ \frac{1}{\lambda}=b(T-T_k) \\ b=\frac{\xi}{\lambda_\infty}, \text{ and } T_k=T_\infty-\frac{1}{\xi} \end{array}\right\} \quad \ldots \quad (7)$$

where b and T_k are constants and their values depend on the reference state and thermal properties of the fluid, i.e. on ξ.

Let us introduce the following similarity transformations and parameters:

$$u=u_\infty f'(\eta),\ \eta=y\sqrt{\frac{u_\infty}{v_\infty x}}$$

$$v=\frac{1}{2}\sqrt{\frac{u_\infty v_\infty}{x}}[\eta f'(\eta)-f(\eta)]$$

$$\theta=\frac{T-T_\infty}{T_w-T_\infty},\ N=u_\infty\left(\frac{u_\infty}{v_\infty x}\right)^{\frac{1}{2}}g(\eta)$$

Using the above transformations the equation of continuity (1) is satisfied identically and rest of the equations (2), (3) and (4) respectively reduced to canonical form as:

$$\left[1-K\left(\frac{\theta-\theta_r}{\theta_r}\right)\right]f'''=\frac{\theta' f''}{\theta-\theta_r}+\left(\frac{ff''}{2}+Kg'\right)\left(\frac{\theta-\theta_r}{\theta_r}\right)$$
$$+\left(\frac{Gr}{Re_x^2}\theta-\frac{M^2}{Re}(f'-1)\right)\left(\frac{\theta-\theta_r}{\theta_r}\right) \qquad (8)$$

$$g''=\frac{1}{G}(2g+f'')+\frac{1}{\Delta}(f'g-fg') \qquad (9)$$

$$\theta''=\frac{\theta'^2}{(\theta-\theta_k)}+\frac{\Pr}{2}\left(\frac{\theta-\theta_k}{\theta_k}\right)f\theta'+\Pr\left(\frac{\theta-\theta_k}{\theta_k}\right)f'\theta$$
$$+\Pr Ec\left(K-\frac{\theta_r}{\theta-\theta_r}\right)\left(\frac{\theta-\theta_k}{\theta_k}\right)f''^2+$$
$$\Pr\frac{M^2}{Re}Ec\left(\frac{\theta-\theta_k}{\theta_k}\right)(f'-1)^2 \qquad (10)$$

The boundary conditions finally become:

$$\left.\begin{array}{l} f'=\alpha f'', f=0, \theta=1+\beta\theta', g=-\frac{1}{2}f'' \text{ at } \eta=0 \\ f'\to 1, \theta\to 0, g\to 0 \ \text{ as } \eta\to\infty \end{array}\right\} \qquad (11)$$

where,

$\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = \frac{1}{\delta(T_w - T_\infty)}$ and $\theta_k = \frac{T_k - T_\infty}{T_w - T_\infty} = \frac{1}{\xi(T_w - T_\infty)}$ are dimensionless reference temperature corresponding to viscosity and thermal conductivity respectively. It is to be noted that these values are negative for liquids and positive for gases when $((T_w - T_\infty))$ is positive (Lai and Kulacki [13]).

Here the dimensionless parameters are defined as:

$G = \frac{c\gamma}{\kappa \nu_\infty}$ is the micro-rotation parameter

$K = \frac{\kappa}{\mu_\infty}$ is the coupling constant parameter

$\Delta = \frac{\gamma}{\mu_\infty j}$ is the material constant

$M = \left(\frac{\sigma}{\mu_\infty}\right)^{\frac{1}{2}} B_0 x$ is the Hartmann number

$\Pr = \frac{\mu_\infty C_p}{\lambda_\infty}$ is the Prandtl number

$Ec = \frac{c^2 x^2}{C_p (T_w - T_\infty)}$ is the Eckert number

$Gr = \frac{g\beta(T_w - T_\infty)x^3}{\nu_\infty^2}$ is the Grashof number

$\alpha = L\frac{u_\infty}{\nu_\infty}$ is the velocity slip parameter

$\beta = D\frac{u_\infty}{\nu_\infty}$ is the thermal slip parameter

3. RESULTS AND DISCUSSION

The systems of differential equations (8) to (10) together with the boundary conditions (11) are solved numerically by applying shooting method, an efficient numerical technique in conjunction with forth order Runge-Kutta method which is solved by developing suitable codes for MATLAB. The numerical values of different parameters are taken as Re=.1, M = .5, Pr = .7, Ec = .01, $\theta_r = -10$, $\theta_k = -10$, G = 1, $\Delta = .5$, K = .01, β =.1, α = .1, Gr = 1 unless otherwise stated.

The graphical representation of velocity profile, temperature profile and micro-rotation profile for various parameters are shown in figure 2 to figure 10. The graphs of velocity profile are shown in figure 2 to 4. In figure 2 and figure 3 we have seen the effects of viscosity parameter θ_r and Hartmann number M on the velocity profile. It is clearly seen that as viscosity parameter θ_r and Hartmann number M increase velocity of the fluid decreases. Physically, Increase in viscous force leads to increase of resistance to the relative motion of the different layers of fluid flow therefore velocity of the fluid decreases. Similarly, when M increases, this also increases the resistive type force i.e. Lorentz force which opposes the flow. It is found in figure 4 that due to increase in velocity slip parameter α velocity of the fluid increases.

Figure 5 to figure 7 represent the graphs of temperature profile for different values of viscosity parameter θ_r, Hartmann number M and thermal conductivity parameter θ_k. From figure 5 and figure 6 it is seen that the temperature of the fluid increases with the increasing values of viscosity parameter θ_r and Hartmann number M . Due to the increase of viscous force the fluid experiences resistance by increasing the friction between its layers and thus thermal boundary layer increases resulting the temperature increases. It is seen in figure 6 that. Figure 7 indicates that the fluid temperature decreases with the increasing values of thermal conductivity parameter θ_k. It is due to the fact that increase in thermal conduction enhances the transportation of heat from a hot region to an adjacent colder region. Since temperature within the boundary layer is more than the outside so temperature becomes less.

The variations of micro-rotation profile for various values of viscosity parameter θ_r, Hartmann number M and velocity slip parameter α shown in figure 8 to figure 10. It is seen in figure 8 and figure 9 that micro-rotation of the fluid elements increases when the viscosity parameter θ_r and Hartmann number M increase. Due to the increase of viscous force and Lorentz force temperature of the fluid increases so molecules get released from their bonds holding them as a result rotation of the fluid elements increases. Figure 10 shows that as the velocity slip parameter α increases micro-rotation of the fluid elements decreases.

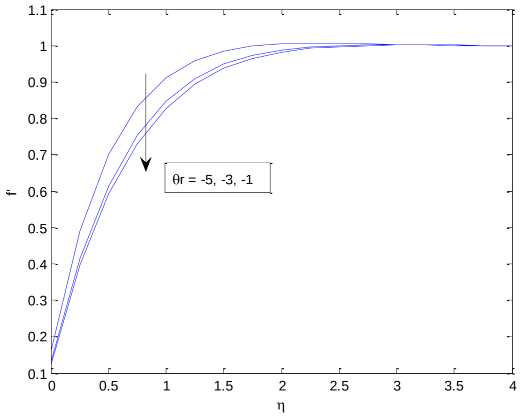

Figure 2:Velocity profile for different θ_r

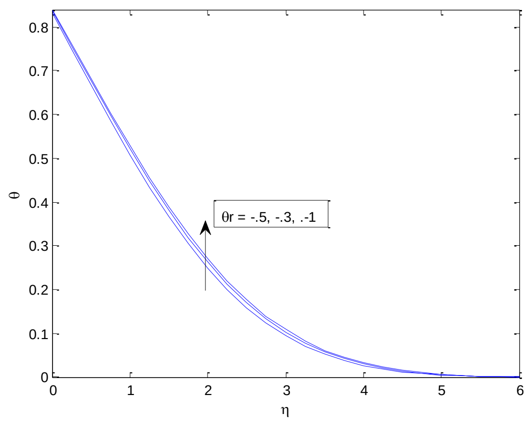

Figure 5: Temperature profile for different θ_r

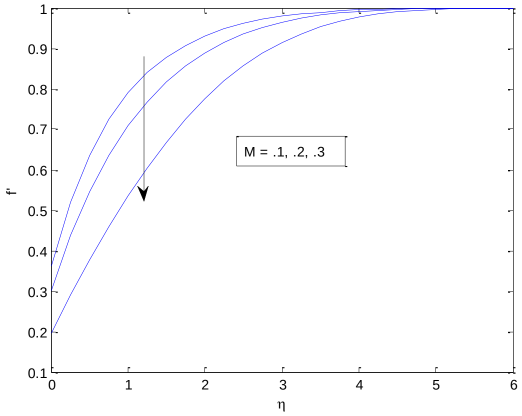

Figure 3:Velocity profile for different M

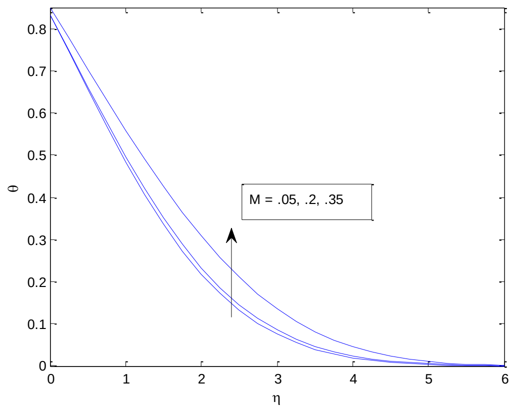

Figure 6: Temperature profile for different M

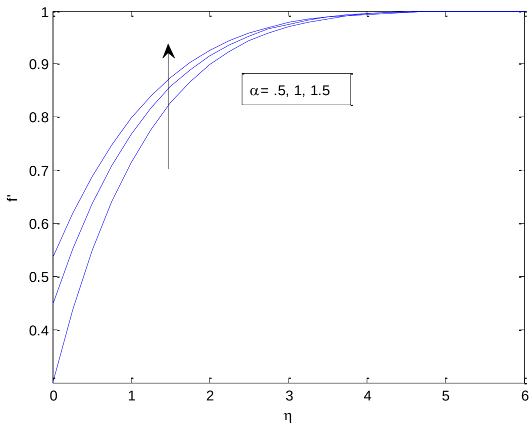

Figure 4:Velocity profile for different α

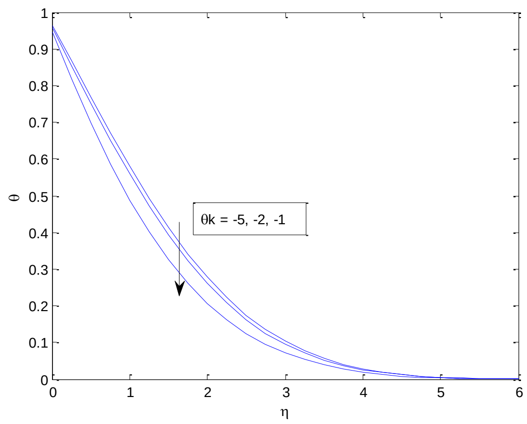

Figure 7: Temperature profile for different θ_k

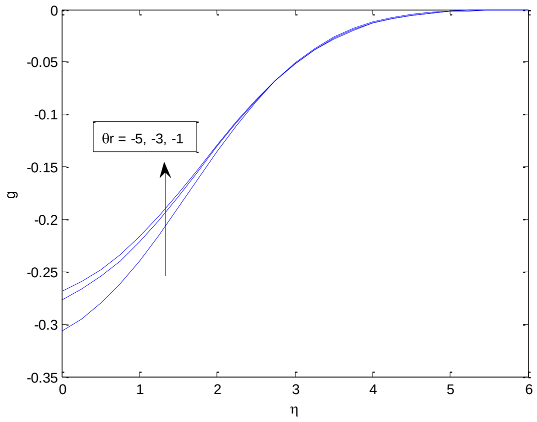

Figure 8: Micro-rotation Profile for different θ_r

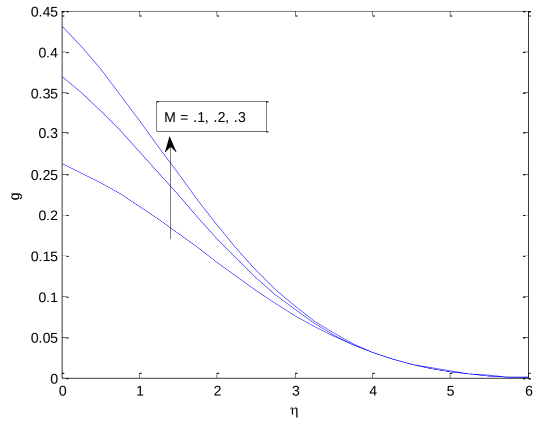

Figure 9: Micro-rotation Profile for different M

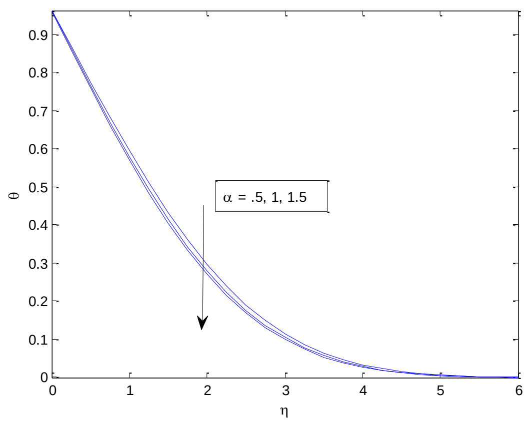

Figure 10: Micro-rotation Profile for different α

4. CONCLUSION

In this study, an investigation has been carried out on the effects of varriable viscosity and thermal conductivity on steady MHD slip flow of a micropolar fluid over a vertical plate.

The following conclusions can be drawn from the above study:

1. Due to the increase of viscosity velocity of the fluid decreases and temperature and micro-rotation of the fluid increases.
2. Increasing values of Hartmann number M enhance the temperature and micro-rotation of the fluid elements but reduce the velocity.
3. Temperature decreases when thermal conductivity increases.
4. Velocity increases and micro-rotation decreases with the increase of velocity slip parameter α.

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