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RW-CLOSED MAPS AND RW-OPEN MAPS IN TOPOLOGICAL SPACES

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Abstract: In this paper we introduce rw-closed map from a topological space X to a topological space Y as the image of every closed set is rw-closed and also we prove that the composition of two rw-closed maps need not be rw-closed map. We also obtain some properties of rw-closed maps.
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1. INTRODUCTION

Generalized closed mappings were introduce and studied by Malghan[5].wg-closed maps and rwg-closed maps were introduced and studied by Nagaveni[6].Regular closed maps,gpr-closed maps and rg-closed maps have been introduced and studied by Long[4], Gnanambal[3] and Arockiarani[1] respectively.

In this paper, a new class of maps called regular weakly closed maps (briefly, rw-closed) maps have been introduced and studied their relations with various generalized closed maps. We prove that the composition of two rw-closed maps need not be rw-closed map. We also obtain some properties of rw-closed maps.

S.S. Benchalli and R.S Wali [2] introduced new class of sets called regular weakly - closed (briefly rw - closed) sets in topological spaces which lies between the class of all w - closed sets and the class of all regular g - closed sets.

Throughout this paper (X, τ) and (Y, σ) (or simply X and Y) represents the non-empty topological spaces on which no separation axiom are assumed, unless otherwise mentioned. For a subset A of X, cl(A) and int(A) represents the closure of A and interior of A respectively.

2. PRELIMINARIES

In this section we recollect the following basic definitions which are used in this paper.

Definition 2.1 [2]: A subset A of a topological space (X,τ) is called rw-closed (briefly rw-closed) if cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is regular semiopen in X.

Definition 2.2 [7]: A subset A of a topological space (X,τ) is called regular generalized closed (briefly rg-closed) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in X.

Definition 2.3 [9]: A subset A of a topological space (X,τ) is called weakly closed (briefly w-closed) if cl(A)) $\subseteq$ U whenever A $\subseteq$ U and U is semi open in X.

Definition 2.4 [7] :A map f: $(X, \tau) \to (Y, \sigma)$ from a topological space X into a topological space Y is called rg continuous if the inverse image of every closed set inY is rg-closed in X.

Definition 2.5 [9] :A map f: $(X, \tau) \to (Y, \sigma)$ from a topological space X into a topological space Y is called w-continuous if the inverse image of every closed set in Y is w-closed in X.

Definition 2.6 [5]: A map f: $(X, \tau) \to (Y, \sigma)$ is called g-closed if f(F) is g-closed in (Y, σ) for every closed set F of (X, τ).

Definition 2.7 [8]: A map f: $(X, \tau) \to (Y, \sigma)$ is called w-closed if f(F) is w-closed in (Y, σ) for every closed set F of (X, τ).

Definition 2.8 [1]:A map f: $(X, \tau) \to (Y, \sigma)$ is called rg-closed if f(F) is rg-closed in (Y, σ) for every closed set F of (X, τ).

Definition 2.9 [10]:A map f: $(X, \tau) \to (Y, \sigma)$ is called g-open if f(U) is g-open in (Y, σ) for every open set U of (X, τ).

Definition 2.10 [8]:A map f: $(X, \tau) \to (Y, \sigma)$ is called w-open if f(U) w-open in (Y, σ) for every open set U of (X, τ).

Definition 2.11[1]:A map f: $(X, \tau) \to (Y, \sigma)$ is called rg-open if f(U) rg-open in (Y, σ) for every open set U of (X, τ).

3. Rw-closed maps

We introduce the following definition

Definition : 3.1 A map $f : (X, \tau) \to (Y, \sigma)$ is said to be regular weakly (briefly rw-closed) if the image of every closed set in (X, τ) is rw-closed in (Y, σ)

Theorem: 3.2 Every closed map is rw-closed map but not conversely.
Proof: The proof follows from the definitions and fact that every closed set is rw-closed.

Remark: 3.3 The converse of the above theorem need not be true as seen from the following example.
Example : 3.4 Consider X=Y={a,b,c} with topologies $\tau = \{X, \phi, \{c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\},\{a,b\}\}$. Let f: $(X, \tau) \to (Y, \sigma)$ be the identity map. Then this function is rw-closed but not closed as the image of closed set {a,b} in X is {a,b} which is not closed set in Y.

Theorem: 3.5 Every rw-closed map is rg-closed map but not conversely.
Proof: The proof follows from the definitions and fact that every rw-closed set is rg-closed.

Remark: 3.6 The converse of the above theorem need not be true as seen from the following example.

Example : 3.7 Consider X=Y={a,b,c,d} with topologies $\tau = \{X, \phi, \{b\},\{a,b,d\}\}$ and $\sigma = \{Y,\phi, \{a\}, \{b\},\{a,b\},\{a,b,c\}\}$. Let f: $(X,\tau) \to (Y,\sigma)$ be the identity map. Then this function is rg-closed but not rw-closed as the image of closed set {c} in X is {c} which is not rw-closed set in Y.

Theorem: 3.8 Every w-closed map is rw-closed map but not conversely.
Proof: The proof follows from the definitions and fact that every w-closed set is rw-closed.

Remark: 3.9 The converse of the above theorem need not be true as seen from the following example.
Example: **3.10** Consider X=Y= {a,b,c} with topologies $\tau = \{X, \phi, \{c\}\}$ and $\sigma = \{Y,\phi, \{a\}, \{b\},\{a,b\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity map. Then this function is rw-closed but not w-closed as the image of closed set {a,b} in X is {a,b} which is not closed set in Y.
Theorem: 3.11 A map f: $(X,\tau) \to (Y,\sigma)$ is rw-closed if and only if for each subset S of (Y,σ) and each open set U containing $f^{-1}(S) \subset U$, there is a rw-open set of (Y, σ) such that $S \subset V$ and $f^{-1}(V) \subset U$.
Proof: Suppose f is rw-closed. Let $S \subset Y$ and U be an open set of (X,τ) such that $f^{-1}(S) \subset U$. Now X-U is closed set in (X,τ). Since f is rw-closed, f(X-U) is rw-closed set in (Y,σ). Then V=Y- f(X-U) is a rw-open set in (Y,σ). Note that $f^{-1}(S) \subset U$ implies $S \subset V$ and $f^{-1}(V) = X- f^{-1}(f(X-U)) \subset X-(X-U) = U$. That is $f^{-1}(V) \subset U$.

For the converse, let F be a closed set of (X,τ). Then $f^{-1}(f(F)^c) \subset F^c$ and F^c is an open set in (X, τ). By hypothesis, there exists a rw-open set V in (Y,σ) such that $f(F)^c \subset V$ and $f^{-1}(V) \subset F^c$ and so $F \subset (f^{-1}(V))^c$. Hence $V^c \subset f(F) \subset f(((f^{-1}(V))^c) \subset V^c$ which implies $f(V) \subset V^c$. Since V^c is rw-closed, f(F) is rw-closed. That is f(F) rw-closed in (Y,σ) and therefore f is rw-closed.
Remark: 3.12 The composition of two rw-closed maps need not be rw-closed map in general and this is shown by the following example.
Example: 3.13 Consider X=Y= {a,b,c} with topologies $\tau = \{X ,\phi, \{b\},\{a,b\}\}$, $\sigma = \{Y,\phi, \{a\}, \{b\},\{a,b\}\}$, $\eta = \{Z, \phi, \{a\}, \{c\},\{a,c\}\}$. Define f: $(X,\tau) \to (Y,\sigma)$ by f(a) = a, f(b) = b and f(c)=c and g : $(Y,\sigma)\to (Z, \eta)$ be the identity map. Then f and g are rw-closed maps but their composition $g \circ f : (X,\tau) \to (Z, \eta)$ is not rw-closed map because F={c} is closed in (X,τ) but $g \circ f(\{a\}) = g(f(\{c\})) = g(\{c\}) = \{c\}$ which is not rw-closed in (Z, η).
Theorem: 3.14 If $f : (X, \tau) \to (Y, \sigma)$ is closed map and $g : (Y,\sigma)\to (Z, \eta)$ is rw-closed map, then the composition $g \circ f : (X,\tau) \to (Z, \eta)$ is rw-closed map.
Proof: Let F be any closed set in (X,τ). Since f is closed map, f (F) is closed set in (Y,σ). Since g is rw-closed map, g (f (F)) is rw-closed set in (Z, η). That is $g \circ f(F) = g(f(F))$ is rw-closed and hence $g \circ f$ is rw-closed map.
Remark: 3.15 If $f : (X, \tau) \to (Y, \sigma)$ is rw-closed map and $g : (Y,\sigma)\to (Z, \eta)$ is closed map, then the composition need not be rw-closed map as seen from the following example.
Example: 3.16 Consider X=Y=Z= {a,b,c} with topologies $\tau = \{X ,\phi, \{b\},\{a,b\}\}$, $\sigma = \{Y,\phi, \{a\}, \{b\},\{a,b\}\}$, $\eta = \{Z, \phi, \{a\}, \{c\},\{a,c\}\}$. Define f: $(X,\tau) \to (Y,\sigma)$ by f (a) = a, f(b) = b and f(c)=c and g : $(Y,\sigma)\to (Z, \eta)$ be the identity map. Then f is rw-closed map and g is a closed map but their composition $g \circ f : (X,\tau) \to (Z, \eta)$ is not rw-closed map since for the closed set {c}in (X,τ) but $g \circ f(\{c\}) = g(f(\{c\})) = g(\{c\}) = \{c\}$ which is not rw-closed in (Z, η).
Theorem: 3.17 Let (X,τ), (Z,η) be topological spaces and (Y,σ) be topological spac where every rw-closed subset is closed. Then the composition $g \circ f : (X,\tau) \to (Z, \eta)$ of the rw-closed maps f: $(X,\tau) \to (Y,\sigma)$ and g : $(Y,\sigma)\to (Z, \eta)$ is rw-closed.
Proof: Let A be a closed set of (X,τ). Since f is rw-closed, f (A) is rw-closed in (Y,σ). Then by hypothesis f (A) is closed. Since g is rw-closed, g (f (A)) is rw-closed in (Z,η) and $g (f (A)) = g \circ f (A)$. Therefore $g \circ f$ is rw-closed.
Theorem: 3.18 If f: $(X,\tau) \to (Y,\sigma)$ is g-closed, g: $(Y,\sigma)\to (Z,\eta)$ be rw-closed and (Y,σ) is $T_{1/2}$ – space then their composition $g \circ f$: $(X,\tau) \to (Z,\eta)$ is rw-closed map.
Proof: Let A be a closed set of (X,τ). Since f is g-closed, f (A) is g-closed in (Y,σ). Since g is rw-closed, g (f (A)) is rw-closed in (Z,η) and $g (f (A)) = g \circ f (A)$.Therefore $g \circ f$ is rw-closed.

Theorem: 3.19 Let **f**: $(X,\tau) \to (Y,\sigma)$ and g: $(Y,\sigma)\to (Z,\eta)$ be two mappings such that their composition $g \circ f: (X,\tau) \rightarrow (Z,\eta)$ be rw-closed mapping. Then the following statements are true.

i) If f is continuous and surjective, then g is rw-closed

ii) If g is rw-irresolute and injective, then f is rw-closed.

iii) If f is g-continuous, surjective and (X,τ) is a $T_{1/2}$ – space, then g is rw-closed.

Proof: i) Let A be a closed set of (Y,σ). Since f is continuous, $f^{-1}(A)$ is closed in $g \circ f(f^{-1}(A))$ is rw-closed in (Z,η). That is g (A) is rw-closed in (Z,η), since f is surjective. Therefore g is rw-closed.

ii) Let B be a closed set of (X,τ). Since $g \circ f$ is rw-closed, $g \circ f(B)$ is rw-closed in (Z,η). Since g is rw-irresolute, $g^{-1}(g \circ f(B))$ is rw-closed set in (Y,σ).That is f (B) is rw-closed in (Y,σ), since f is injective. Therefore f is rw-closed.

iii) Let c be a closed set of (Y,σ). Since f is g-continuous, $f^{-1}(c)$ is g-closed set in (X,τ). Since (X,τ) is a $T_{1/2}$ -space, $f^{-1}(c)$ is closed set in (X,τ). Since $g \circ f$ is rw-closed $(g \circ f)(f^{-1}(c))$ is rw-closed in (Z,η). That is g(c) is rw-closed in (Z,η), since f is surjective. Therefore g is rw-closed.

4. Rw-open maps

Definition: 4.1 A map $f: (X,\tau) \rightarrow (Y,\sigma)$ is called a rw-open map if the image f (A) is rw-open in (Y,σ) for each open set A in (X,τ)

Theorem: 4.2 For any bijection map $f: (X,\tau) \rightarrow (Y,\sigma)$ the following statements are equivalent.

i) $f^{-1} : (Y,\sigma) \rightarrow (X,\tau)$ is rw-continuous

ii) f is rw-open map and

iii) f is rw-closed map.

Proof: (i) $\Rightarrow$ (ii) Let U be an open set of (X,τ). By assumption, $(f^{-1})^{-1}(U) = f(U)$ is rw-open in (Y,σ) and so f is rw-open.

(ii) $\Rightarrow$ (iii) Let F be a closed set of (X,τ). Then F^c is open set in (X,τ). By assumption $f(F^c)$ is rw-open in (Y,σ) . That is $f(F^c) = f(F)^c$ is rw-open in (Y,σ) and therefore f(F) is rw-closed in (Y,σ). Hence F is rw-closed.

(iii) $\Rightarrow$ (i) Let F be a closed set of (X,τ). By assumption, f (F) is rw-closed in (Y,σ). But $f(F) = (f^{-1})^{-1}(F)$ and therefore f^{-1} is continuous.

Theorem: 4.3 A map $f: (X,\tau) \rightarrow (Y,\sigma)$ is rw-open if and only if for any subset S of (Y,σ) and any closed set of (X,τ) containing $f^{-1}(S)$, there exists a rw-closed set K of (Y,σ) containing S such that $f^{-1}(K) \subset F$.

Proof: Suppose f is rw-open map. Let $S \subset Y$ and F be a closed set of (X,τ) such that $f^{-1}(S) \subset F$. Now X-F is an open set in (X,τ). Since f is rw-open map, f (X-F) is rw-open set in (Y,σ). Then K=Y- f (X-F) is a rw-closed set in (Y,σ). Note that $f^{-1}(S) \subset F$ implies $S \subset K$ and $f^{-1}(K) = X- f^{-1}(X-F) \subset X-(X-F) = F$. That is $f^{-1}(K) \subset F$.

For the converse let U be an open set of (X,τ). Then $f^{-1}((f(U))^c) \subset U^c$ and U^c is a closed set in (X,τ). By hypothesis, there exists a rw-closed set K of (Y,σ) such that $(f(U))^c \subset K$ and $f^{-1}(K) \subset U^c$ and so $U \subset (f^{-1}(K))^c$. Hence $K^c \subset f(U) \subset f((f^{-1}(K)))^c$ which implies $f(U) = K^c$. Since K^c is a rw-open, f(U) is rw-open in (Y,σ) and therefore f is rw-open map.

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