

Intuitionistic Fuzzy Semipre Generalized Connected Spaces

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Abstract : In this paper, we introduce the notion of intuitionistic fuzzy semipre generalized connected space, intuitionistic fuzzy semipre generalized super connected space and intuitionistic fuzzy semipre generalized extremally disconnected spaces. We investigate some of their properties.

Keywords : Intuitionistic fuzzy topology, intuitionistic fuzzy semipre generalized connected space, intuitionistic fuzzy semipre generalized super connected space and intuitionistic fuzzy semipre generalized extremally disconnected spaces

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1. INTRODUCTION

In 1965, Zadeh [11] introduced fuzzy sets and in 1968, Chang [2] introduced fuzzy topology. After the introduction of fuzzy set and fuzzy topology, several authors were conducted on the generalization of this notion. The notion of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. 1997, Coker [3] introduced the concept of intuitionistic fuzzy topological spaces. In this paper, we introduce the notion of intuitionistic fuzzy semipre generalized connected space, intuitionistic fuzzy semipre generalized super connected space and intuitionistic fuzzy semipre generalized extremally disconnected spaces. And study some of their properties.

2. PRELIMINARIES

Definition 2.1:[1] Let X be a non-empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ where the functions $\mu_A: X \rightarrow [0, 1]$ and $\nu_A: X \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by $\text{IFS}(X)$, the set of all intuitionistic fuzzy sets in X .

Definition 2.2:[1] Let A and B be IFSs of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$. Then

1. $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
2. $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
3. $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$
4. $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$

$$5. \quad A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$. The intuitionistic fuzzy sets $0_{\sim} = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1_{\sim} = \{ \langle x, 1, 0 \rangle / x \in X \}$ are respectively the empty set and the whole set of X .

Definition 2.3:[3] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

1. $0_{\sim}, 1_{\sim} \in \tau$
2. $G_1 \cap G_2 \in \tau$, for every $G_1, G_2 \in \tau$
3. $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X . The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X .

Definition 2.4:[3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X . Then

1. $\text{int}(A) = \cup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \}$
2. $\text{cl}(A) = \cap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}$
3. $\text{cl}(A^c) = (\text{int}(A))^c$
4. $\text{int}(A^c) = (\text{cl}(A))^c$

Definition 2.5:[10] An IFS A of an IFTS (X, τ) is an

1. intuitionistic fuzzy semipre closed set (IFSPCS for short) if there exists an IFPCS B such that $\text{int}(B) \subseteq A \subseteq B$

2. intuitionistic fuzzy semipre open set (IFSPOS for short) if there exists an IFPOS B such that $B \subseteq A \subseteq \text{cl}(B)$

Definition 2.6:[4] Let A be an IFS in an IFTS (X, τ) . Then

1. $\text{spint}(A) = \cup \{ G / G \text{ is an IFSPOS in } X \text{ and } G \subseteq A \}$

2. $\text{spcl}(A) = \cap \{ K / K \text{ is an IFSPCS in } X \text{ and } A \subseteq K \}$

Note that for any IFS A in (X, τ) , we have $\text{spcl}(\text{Ac}) = (\text{spint}(A))^c$ and $\text{spint}(\text{Ac}) = (\text{spcl}(A))^c$.

Definition 2.7:[7] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy semipre generalized closed set (IFSPGCS for short) if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSOS in (X, τ) .

The family of all IFSPGCSs of an IFTS (X, τ) is denoted by $\text{IFSPGC}(X)$. Every IFCS and IFSPCS is an IFSPGCS but the converses are not true in general.

Definition 2.8:[5] The complement Ac of an IFSPGCS A in an IFTS (X, τ) is called an intuitionistic fuzzy semipre generalized open set (IFSPGOS for short) in X .

The family of all IFSPGOSs of an IFTS (X, τ) is denoted by $\text{IFSPGO}(X)$. Every IFOS and IFSPOS is an IFSPGOS but the converses are not true in general.

Definition 2.9:[9] Let A be an IFS in an IFTS (X, τ) . Then semipre generalized interior of A ($\text{spgint}(A)$ for short) and semipre generalized closure of A ($\text{spgcl}(A)$ for short) are defined by

1. $\text{spgint}(A) = \cup \{ G / G \text{ is an IFSPGOS in } X \text{ and } G \subseteq A \}$

2. $\text{spgcl}(A) = \cap \{ K / K \text{ is an IFSPGCS in } X \text{ and } A \subseteq K \}$

Note that for any IFS A in (X, τ) , we have $\text{spgcl}(\text{Ac}) = (\text{spgint}(A))^c$ and $\text{spgint}(\text{Ac}) = (\text{spgcl}(A))^c$.

Definition 2.10:[5] An IFTS (X, τ) is said to be an intuitionistic fuzzy semipre $T^*_{1/2}$ space (IFSP $T^*_{1/2}$ space for short) if every IFSPGCS is an IFCS in (X, τ) .

Definition 2.11:[8] A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy semipre generalized continuous (IFSPG continuous for short) mappings if $f^{-1}(V)$ is an IFSPGCS in (X, τ) for every IFCS V of (Y, σ) .

Definition 2.12:[8] A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic fuzzy semipre generalized irresolute (IFSPG irresolute) mapping if $f^{-1}(V)$ is an IFSPGCS in (X, τ) for every IFSPGCS V of (Y, σ) .

Definition 2.13:[4] Two IFSs A and B are said to be q -coincident ($A q B$ in short) if and only if there exists an

element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.14:[4] Two IFSs A and B are said to be not q -coincident ($A \text{ qc } B$ in short) if and only if $A \subseteq B^c$.

Definition 2.15:[6] An IFTS (X, τ) is said to be an intuitionistic fuzzy C_5 -connected (IFC $_5$ -connected for short) space if the only IFSs which are both intuitionistic fuzzy open and intuitionistic fuzzy closed are 0_{\sim} and 1_{\sim} .

3. INTUITIONISTIC FUZZY SEMIPRE GENERALIZED CONNECTED SPACES

Definition 3.1: An IFTS (X, τ) is said to be an intuitionistic fuzzy semipre generalized connected space (IFSPG connected space for short) if the only IFSs which are both an IFSPGOS and an IFSPGCS are 0_{\sim} and 1_{\sim} .

Theorem 3.2: Every IFSPG connected space is IFC $_5$ -connected but not conversely.

Proof: Let (X, τ) be an IFSPG connected space. Suppose (X, τ) is not IFC $_5$ -connected, then there exists a proper IFS A which is both an IFOS and an IFCS in (X, τ) . That is, A is both an IFSPGOS and an IFSPGCS in (X, τ) . This implies that (X, τ) is not IFSPG connected. This is a contradiction. Therefore (X, τ) must be an IFC $_5$ -connected space.

Example 3.3: Let $X = \{a, b\}$ and $G = \langle x, (0.5, 0.6), (0.5, 0.4) \rangle$. Then $\tau = \{0_{\sim}, G, 1_{\sim}\}$ is an IFT on X . Then X is an IFC $_5$ -connected space but not IFSPG connected, since the IFS $A = \langle x, (0.5, 0.7), (0.5, 0.3) \rangle$ in X is both an IFSPGCS and an IFSPGOS in X .

Theorem 3.4: An IFTS (X, τ) is an IFSPG connected space if and only if there exists no non-zero IFSPGOSs A and B in (X, τ) such that $B = \text{Ac}$, $B = (\text{spcl}(A))^c$, $A = (\text{spcl}(B))^c$.

Proof: Necessity: Assume that there exist IFSs A and B such that $A \neq 0_{\sim} \neq B$, $B = \text{Ac}$, $B = (\text{spcl}(A))^c$, $A = (\text{spcl}(B))^c$. Since $(\text{spcl}(A))^c$ and $(\text{spcl}(B))^c$ are IFSPGOSs in (X, τ) , A and B are IFSPGOSs in (X, τ) . This implies (X, τ) is not IFSPG connected, which is a contradiction. Therefore there exists no non-zero IFSPGOSs A and B in (X, τ) such that $B = \text{Ac}$, $B = (\text{spcl}(A))^c$, $A = (\text{spcl}(B))^c$.

Sufficiency: Let A be both an IFSPGOS and an IFSPGCS in (X, τ) such that $1_{\sim} \neq A \neq 0_{\sim}$. Now by taking $B = \text{Ac}$, we obtain a contradiction to our hypothesis. Hence (X, τ) is an IFSPG connected space.

Theorem 3.5: Let (X, τ) be an IFSP $T^*_{1/2}$ space, then the following statements are equivalent:

- (i) (X, τ) is an IFSPG connected space,

(ii) (X, τ) is an IFC5-connected space.

Proof: (i) \Rightarrow (ii) is obvious by Theorem 3.2.

(ii) \Rightarrow (i) Let (X, τ) be an IFC5-connected space. Suppose (X, τ) is not IFSPG connected, then there exists a proper IFS A in (X, τ) which is both an IFSPGOS and an IFSPGCS in (X, τ) . But since (X, τ) is an IFSPT*1/2 space, A is both an IFOS and an IFCS in (X, τ) . This implies that (X, τ) is not IFC5-connected, which is a contradiction to our hypothesis. Therefore (X, τ) must be an IFSPG connected space.

Theorem 3.6: If $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IFSPG continuous surjection and (X, τ) is an IFSPG connected space, then (Y, σ) is an IFC5-connected space.

Proof: Let (X, τ) be an IFSPG connected space. Suppose (Y, σ) is not IFC5-connected, then there exists a proper IFS A which is both an IFOS and an IFCS in (Y, σ) . Since f is an IFSPG continuous mapping, $f^{-1}(A)$ is both an IFSPGOS and an IFSPGCS in (X, τ) . But this is a contradiction to our hypothesis. Hence (Y, σ) must be an IFC5- connected space.

Theorem 3.7: If $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IFSPG irresolute surjection and (X, τ) is an IFSPG connected space, then (Y, σ) is also an IFSPG connected space.

Proof: Suppose (Y, σ) is not an IFSPG connected space, then there exists a proper IFS A such that A is both an IFSPGOS and an IFSPGCS in (Y, σ) . Since f is an IFSPG irresolute surjection, $f^{-1}(A)$ is both an IFSPGOS and an IFSPGCS in (X, τ) . But this is a contradiction to our hypothesis. Hence (Y, σ) must be an IFSPG connected space.

Definition 3.8: An IFTS (X, τ) is IFSPG connected between two IFSs A and B if there is no IFSPGOS E in (X, τ) such that $A \subseteq E$ and $E \subset B$.

Example 3.9: Let $X = \{a, b\}$ and $G = \langle x, (0.5, 0.4), (0.5, 0.6) \rangle$. Then $\tau = \{0\sim, G, 1\sim\}$ is an IFT on X . Let $A = \langle x, (0.5, 0.4), (0.5, 0.3) \rangle$ and $B = \langle x, (0.5, 0.4), (0.5, 0.5) \rangle$ be two IFSs in X . Hence (X, τ) is IFSPG connected between the IFSs A and B .

Theorem 3.10: If an IFTS (X, τ) is IFSPG connected between two IFSs A and B , then it is IFC5-connected between A and B but the converse may not be true in general.

Proof: Suppose (X, τ) is not IFC5-connected between A and B , then there exists an IFOS E in (X, τ) such that $A \subseteq E$ and $E \subset B$. Since every IFOS is an IFSPGOS, there exists an IFSPGOS E in (X, τ) such that $A \subseteq E$ and $E \subset B$. This implies (X, τ) is not IFSPG connected between A and B , a contradiction to our hypothesis. Therefore (X, τ) must be IFC5-connected between A and B .

Example 3.11: Let $X = \{a, b\}$ and $G = \langle x, (0.5, 0.4), (0.5, 0.6) \rangle$. Then $\tau = \{0\sim, G, 1\sim\}$ is an IFT on X . Let $A = \langle x, (0.4, 0.4), (0.6, 0.6) \rangle$ and $B = \langle x, (0.3, 0.3), (0.4, 0.4) \rangle$ be two IFSs in X . Then X is IFC5-connected between A and B , since there exists no IFOS E in X such that $A \subseteq E$ and $E \subset B$. But it is not IFSPG connected

between the two IFS A and B , since there exists an IFSPGOS $E = \langle x, (0.4, 0.4), (0.5, 0.5) \rangle$ in X such that $A \subseteq E$ and $E \subset B$.

Theorem 3.12: If an IFTS (X, τ) is IFSPG connected between A and B and $A \subseteq A_1, B \subseteq B_1$, then (X, τ) is IFSPG connected between A_1 and B_1 .

Proof: Suppose that (X, τ) is not IFSPG connected between A_1 and B_1 , then by Definition 3.8, there exists an IFSPGOS E in (X, τ) such that $A_1 \subseteq E$ and $E \subset B_1$. This implies $E \subseteq B_1$. $A_1 \subseteq E$ implies $A \subseteq A_1 \subseteq E$. That is $A \subseteq E$. Now let us prove that $E \subseteq B$, that is $E \subset B$. Suppose that $E \not\subset B$, then by Definition 2.13, there exists an element x in X such that $\mu_E(x) > \nu_B(x)$ and $\nu_E(x) < \mu_B(x)$. Therefore $\mu_E(x) > \nu_B(x) > \nu_{B_1}(x)$ and $\nu_E(x) < \mu_B(x) < \mu_{B_1}(x)$, since $B \subseteq B_1$. Hence $\mu_E(x) > \mu_{B_1}(x)$ and $\nu_E(x) < \nu_{B_1}(x)$. Thus $E \not\subset B_1$. That is $E \not\subset B_1$, which is a contradiction. Therefore $E \subset B$. That is $E \subseteq B$. Hence (X, τ) is not IFSPG connected between A and B , which is a contradiction to our hypothesis. Thus (X, τ) must be IFSPG connected between A_1 and B_1 .

Theorem 3.13: Let (X, τ) be an IFTS and A and B be IFSs in (X, τ) . If $A \subset B$, then (X, τ) is IFSPG connected between A and B .

Proof: Suppose (X, τ) is not IFSPG connected between A and B . Then there exists an IFSPGOS E in (X, τ) such that $A \subseteq E$ and $E \subset B$. This implies that $A \subseteq B$. That is $A \subset B$. But this is a contradiction to our hypothesis. Therefore (X, τ) is must be IFSPG connected between A and B .

Definition 3.14: An IFSPGOS A is an intuitionistic fuzzy regular semipre generalized open set (IFRSPGOS for short) if $A = \text{spgint}(\text{spgcl}(A))$. The complement of an IFRSPGOS is called an intuitionistic fuzzy regular semipre generalized closed set (IFRSPGCS for short).

Definition 3.15: An IFTS (X, τ) is called an intuitionistic fuzzy semipre generalized (IFSPG for short) super connected space if there exists no proper IFRSPGOS in (X, τ) .

Theorem 3.16: Let (X, τ) be an IFTS. Then the following statements are equivalent:

- (i) (X, τ) is an IFSPG super connected space
- (ii) For every non-zero IFSPGOS A , $\text{spgcl}(A) = 1\sim$
- (iii) For every IFSPGCS A with $A \neq 1\sim$, $\text{spgint}(A) = 0\sim$
- (iv) There exists no IFSPGOSs A and B in (X, τ) such that $A \neq 0\sim \neq B, A \subseteq B$
- (v) There exists no IFSPGOSs A and B in (X, τ) such that $A \neq 0\sim \neq B, B = (\text{spgcl}(A))^c, A = (\text{spgcl}(B))^c$
- (vi) There exists no IFSPGCSs A and B in (X, τ) such that $A \neq 0\sim \neq B, B = (\text{spgint}(A))^c, A = (\text{spgint}(B))^c$

Proof: (i) \Rightarrow (ii) Assume that there exists an IFSPGOS A in (X, τ) such that $A \neq 0_{\sim}$ and $\text{spgcl}(A) \neq 1_{\sim}$. Now let $B = \text{spgint}(\text{spgcl}(A))c$. Then B is a proper IFRSPGOS in (X, τ) , which is contradiction. Therefore $\text{spgcl}(A) = 1_{\sim}$.

(ii) \Rightarrow (iii) Let $A \neq 1_{\sim}$ be an IFSPGCS in (X, τ) . If $B = Ac$, then B is an IFSPGOS in (X, τ) with $B \neq 0_{\sim}$. Hence $\text{spgcl}(B) = 1_{\sim}$. This implies $(\text{spgcl}(B))c = 0_{\sim}$. That is $\text{spgint}(Bc) = 0_{\sim}$. Hence $\text{spgint}(A) = 0_{\sim}$.

(iii) \Rightarrow (iv) Suppose A and B be two IFSPGOSs in (X, τ) such that $A \neq 0_{\sim} \neq B$ and $A \subseteq Bc$. Then Bc is an IFSPGCS in (X, τ) and $B \neq 0_{\sim}$ implies $Bc \neq 1_{\sim}$. By hypothesis $\text{spgint}(Bc) = 0_{\sim}$. But $A \subseteq Bc$. Therefore $0_{\sim} \neq A = \text{spgint}(A) \subseteq \text{spgint}(Bc) = 0_{\sim}$, which is a contradiction. Therefore (iv) is true.

(iv) \Rightarrow (i) Suppose $0_{\sim} \neq A \neq 1_{\sim}$ be an IFRSPGOS in (X, τ) . If we take $B = (\text{spgcl}(A))c$, we get $B \neq 0_{\sim}$, since if $B = 0_{\sim}$ then this implies $(\text{spgcl}(A))c = 0_{\sim}$. That is $\text{spgcl}(A) = 1_{\sim}$. Hence $A = \text{spgint}(\text{spgcl}(A)) = \text{spgint}(1_{\sim}) = 1_{\sim}$, which is a contradiction. Therefore $B \neq 0_{\sim}$ and $A \subseteq Bc$. But this is a contradiction to (iv). Therefore (X, τ) must be an IFSPG super connected space.

(i) \Rightarrow (v) Suppose A and B be two IFSPGOSs in (X, τ) such that $A \neq 0_{\sim} \neq B$ and $B = (\text{spgcl}(A))c$, $A = (\text{spgcl}(B))c$. Now we have $\text{spgint}(\text{spgcl}(A)) = \text{spgint}(Bc) = (\text{spgcl}(B))c = A$, $A \neq 0_{\sim}$ and $A \neq 1_{\sim}$, since if $A = 1_{\sim}$, then $1_{\sim} = (\text{spgcl}(B))c \Rightarrow \text{spgcl}(B) = 0_{\sim} \Rightarrow B = 0_{\sim}$. Therefore $A \neq 1_{\sim}$. That is, A is a proper IFRSPGOS in (X, τ) , which is a contradiction to (i). Hence (v) is true.

(v) \Rightarrow (i) Let A be an IFSPGOS in (X, τ) such that $A = \text{spgint}(\text{spgcl}(A))$, $0_{\sim} \neq A \neq 1_{\sim}$. Now take $B = (\text{spgcl}(A))c$. In this case we get, $B \neq 0_{\sim}$ and B is an IFSPGOS in (X, τ) . Now $B = (\text{spgcl}(A))c$ and $(\text{spgcl}(B))c = (\text{spgcl}(\text{spgcl}(A))c)c = \text{spgint}(\text{spgcl}(A)) = A$. But this is a contradiction to (v). Therefore (X, τ) must be an IFSPG super connected space.

(v) \Rightarrow (vi) Suppose A and B be two IFSPGCS in (X, τ) such that $A \neq 1_{\sim} \neq B$, $B = (\text{spgint}(A))c$, $A = (\text{spgint}(B))c$. Taking $C = Ac$ and $D = Bc$, C and D become IFSPGOSs in (X, τ) with $C \neq 0_{\sim} \neq D$ and $D = (\text{spgcl}(C))c$, $C = (\text{spgcl}(D))c$, which is a contradiction to (v). Hence (vi) is true.

(vi) \Rightarrow (v) can be proved easily by the similar way as in (v) \Rightarrow (vi).

Definition 3.17: An IFTS (X, τ) is said to be an intuitionistic fuzzy semipre generalized (IFSPG for short) extremally disconnected if the semipre generalized closure of every IFSPGOS in (X, τ) is an IFSPGOS in (X, τ) .

Theorem 3.18: Let (X, τ) be an IFTS space. Then the following statements are equivalent:

(vi) \Rightarrow (v) can be proved easily by the similar way as in (v) \Rightarrow (vi).

Definition 3.17: An IFTS (X, τ) is said to be an intuitionistic fuzzy semipre generalized (IFSPG for short)

) extremally disconnected if the semipre generalized closure of every IFSPGOS in (X, τ) is an IFSPGOS in (X, τ) .

Theorem 3.18: Let (X, τ) be an IFTS space. Then the following statements are equivalent:

- (i) (X, τ) is an IFSPG extremally disconnected
- (ii) For each IFSPGCS A , $\text{spgint}(A)$ is an IFSPGCS
- (iii) For each IFSPGOS A , $\text{spgcl}(A) = (\text{spgcl}(\text{spgcl}(A))c)c$
- (iv) For each pair of IFSPGOSs A and B with $\text{spgcl}(A) = Bc$ implies that $\text{spgcl}(A) = (\text{spgcl}(B))c$

Proof : (i) \Rightarrow (ii) Let A be any IFSPGOS. Then Ac is an IFSPGOS. So (i) implies that $\text{spgcl}(Ac) = (\text{spgint}(A))c$ is an IFSPGOS. Thus $\text{spgint}(A)$ is an IFSPGCS in (X, τ) .

(ii) \Rightarrow (iii) Let A be an IFSPGOS. We have $(\text{spgcl}(\text{spgcl}(A))c)c = (\text{spgcl}(\text{spgint}(Ac)))c$. Since A is an IFSPGOS, Ac is IFSPGCS. So by (ii) $\text{spgint}(Ac)$ is an IFSPGCS. That is $\text{spgcl}(\text{spgint}(Ac)) = \text{spgint}(Ac)$. Hence $(\text{spgcl}(\text{spgint}(Ac)))c = (\text{spgint}(Ac))c = \text{spgcl}(A)$.

(iii) \Rightarrow (iv) Let A and B be any two IFSPGOSs in (X, τ) such that $\text{spgcl}(A) = Bc$. (iii) implies that $\text{spgcl}(A) = (\text{spgcl}(\text{spgcl}(A))c)c = (\text{spgcl}(Bc))c = (\text{spgcl}(B))c$.

(iv) \Rightarrow (i) Let A be any IFSPGOS in (X, τ) . Put $B = (\text{spgcl}(A))c$, then B is an IFSPGOS and $\text{spgcl}(A) = Bc$. Hence by (iv), $\text{spgcl}(A) = (\text{spgcl}(B))c$. Since $\text{spgcl}(B)$ is an IFSPGCS, it follows that $\text{spgcl}(A)$ is an IFSPGOS. This implies that (X, τ) is an IFSPG extremally disconnected space.

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