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Order Size Dependent Trade Credit Study in a Three Echelon Supply Chain Model

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Abstract: In present study, we generalize order linked trade credit policy in three echelon supply chain system where manufacturer, distributor and retailer are involved and manufacturer provide a delay period to distributor and distributor also provide a order linked trade credit policy to his retailers. Whole study is discussed in time dependent production and demand rate. We model a three echelon supply chain system as cost minimization to determine the system's optimal cycle time. In this paper, we determine the optimal cycle time, optimal order quantity and optimal payment time. Finally numerical examples are given to illustrate the result and the managerial insights are also obtained.

Keywords: Time dependent demand, variable production rate, three echelon supply chain, order linked trade credit.

1. INTRODUCTION

Over the last few decades, the supply chain design and management issues have been widely studied. Still, these are attractive research topics, partly because of the relentless drive to lower cost and partly because of improving service quality through efficient information sharing/exchange among different parties involved in the entire supply chain.

Traditionally, inventory models considered the different subsystem in the supply chain independently. With the recent advances in communication and information technologies, the integration of these function are a common phenomena. Moreover due to limited resources, increasing competition and market globalization, enterprises are force to develop supply chain that can respond quickly to customer need with minimum stock and minimum service level. The cooperation between manufacturers and retailers, Ishii et al (1988) considered a three echelon system with one manufacturer, one wholesaler and one retailer respectively. Haq et al (1991) considered a three echelon system with one production facility, several warehouses and several retailers. Woo et al (2001) considered an integrated inventory system where a vendor purchases and process raw materials and delivered the finished items to multiple buyers. Rau et al (2003) developed a multi-echelon inventory model for a deteriorating item and derived an optimal joint total cost from an integrated perspective among the suppliers, the producer and the buyers. We address in this paper a three echelon supply chain with linearly increasing time dependent demand rate, production rate and permissible delay in payments. Under most market behavior, permissible delay in payment can provide economic sense for vendor because of the vendor provides a credit period to the buyer can stimulate demand so as minimize the vendor's own benefits and advantage or minimize total cost. Therefore, the extensive use of trade credit as an alternative has been addressed by Goyal (1985) who stabilized a single item inventory model under permissible delay in payment. Chang (1998) developed an alternative approach to determine the EOQ under the condition of permissible delay in payments. Abad and Jaggi (2003) developed a seller-buyer model with a permissible delay in payments by game theory to determine the optimal unit price and the credit period, considering that the demand rate is a function of retail price. Ouyang et al. (2006) discussed a study on an inventory model for non instantaneous deteriorating items with permissible delay in payments. Goyal et al. (2007) established optimal ordering policies when the supplier provides a progressive interest-payable scheme. Singh et al. (2007) presented an inventory model for perishable with quadratic demand, partial backlogging and permissible delay in payments. Liao (2008) studied deteriorating items under two-level trade credit. Jaber and Goyal (2008) considered channel coordination in a three-level supply chain. They assumed that both demand and supply are certain. Thangam and Uthaykumar (2009) developed two echelon trade credits financing for perishable items in a supply chain when demands on both selling price and credit period. Singh et al (2010) developed an EOQ model with Pareto distribution for deterioration, Trapezoidal type demand and backlogging under trade credit policy. Chen and Bell (2011) investigated a channel that consists of a manufacturer and a retailer where the retailer simultaneously determines the retail price and order quantity while experiencing customer returns and price dependent stochastic demand. They proposed an agreement that includes two buyback prices, one for unsold inventory and one for customer returns and show that this revised returns policy can achieve perfect supply-chain coordination and lead to a win–win situation. Singh et al (2011) developed two warehouse fuzzy inventory models under the condition of permissible delay in payments. Su (2012) presented an optimal replenishment policy for an integrated inventory system with defective items and allowable shortage under trade credit. Singh and Singh (2012) discussed an integrated supply chain model for perishable items with trade credit policy under imprecise environment. Soni (2013) discussed optimal replenishment policies for deteriorating items with stock sensitive demand under two-level trade credit and limited capacity. Yadav et al (2013) analyzed the retailer's optimal policy under inflation in fuzzy environment with trade credit. Omar et al (2013) discussed a just-in-time three-level integrated manufacturing system for linearly time-varying demand process and has taken decreasing time varying demand rate for customer. Chung and Barron (2013) simplified solution procedure for deteriorating items under stock dependent demand and two level trade credits in the supply chain management.

Supplier credit policy offered to the retailer where credit terms are independent of the order quantity. That is, whatever the order quantity is small or large the retailer can take the benefits of payment delay. Under this condition, the effect of stimulating the retailer's demand may reduce. So, the present study will adopt the following assumption to modify the Goyal (1985) model. To encourage to retailer to order a large quantity the supplier may give the trade credit period only for a large order quantity. In other word the retailer requires payment for a small order quantity. For this point Khouja and Mehrez (1996) investigated the effect of supplier credit policies where credit terms are linked to the order quantity. There are so many researchers that have done work in this direction as Jaggi et al. (2008) developed an optimal replenishment decisions policy with credit-linked demand under permissible delay in payments for retailer. Chang et al. (2009) analyzed the optimal pricing and ordering policy for an integrated inventory model when trade credit linked to order quantity. Chang et al. (2010) derived the optimal ordering policies for deteriorating items when a trade credit is linked to order quantity.

In this study we have considered the order linked trade credit concept in three echelon supply chain inventory model where manufacturer also provides a delay period to his distributor and distributor offer order linked delay period to his retailers to promote his sell. The main purpose of this study to discuss this order linked trade credit policy with time dependent demand rate and production rate in three echelon supply chain inventory model.

2. ASSUMPTIONS AND NOTATIONS

The following assumptions and notations are used to the single channel multi-echelon supply chain system with trade credit consideration.

2.1 Assumptions

1. The retailer's ordering quantity from distributor has to be on JIT basis that may require small and frequent replenishment basis and all shipments are of equal basis.
2. Demand rate is variable and time dependent, D = a+bt and Production rate is demand dependent, i.e. P=k D, where a, b $\geq$0, a > b and k>1
3. Manufacturer offers a certain permissible delay period to his distributor and the distributer offers a conditional trade credit to the retailers such as if $Q_r < w$, the trade credit is not permitted. Otherwise, fixed trade credit period *M* is permitted. Hence, if $Q_r < w$, pay $S_d Q_r$ when order is received. If $Q_r \geq w$, pay $S_d Q_r$ *M* time periods after the order is received.
4. During the time the account is not settled, generated sales revenue is deposited in an interest bearing account. When $T \geq M$, the account is settled at T=M, the retailer starts paying for the higher interest charges on the items in stock. When $T \leq M$, the account is settled at T=M and the retailer does not need to pay any interest charges.
5. The ordering cycle times (the time interval in successive orders) are equal for both distribution canters and retailers, that is same as the production cycle time of the manufacturer.
6. There is no repair and replenishment of deteriorated items.
7. Time horizon is infinite. $S_d \geq S_m$, $S_r \geq S_d$, $I_s \geq I_c$, $N \geq M$

2.2 Notations

Manufacturer's parameters

D	annual demand rate such as $D = a + bt$ where a, b≥0 and a>b
P	annual production rate of manufacturer as $P = kD$ where k>1
A_m	fixed production setup cost per lot size
h_m	stock holding cost per unit per year ($ / unit/ year)
τ_m	the transportation cost of a shipment from manufacturer to supplier
$I_{m_1}(t)$	the inventory level that changes with time t during production period
$I_{m_2}(t)$	the inventory level that changes with time t during non-production period
T	common cycle time of production/ordering cycle
S_m	unit selling price per item of good quality
I_m	annual interest rate for calculating the manufacturer's opportunity interest loss due to the delay payment
TAC_m	the annual total relevant cost of the manufacturer

Distributor's parameters

A_d	the distributor ordering cost per shipment
h_d	stock holding cost per unit per year (dollar/unit/year)
τ_{d1}	the transportation cost of receiving a shipment from manufacturer
τ_{d2}	the transportation cost of the distributor of delivering a shipment to retailer
N	distributor's permissible delay period offered by manufacturer to distributor
n	number of shipment per order from manufacturer to distributor, $n \geq 1$
$I_d(t)$	the inventory level that changes with time t during the period T_3
T_3	the replenishment time interval and $T_3 = T / n$
I_0	annual interest rate for calculating the distributor
S_d	unit selling price per item of good quality
Q_d	shipment quantity from manufacturer to distributor in each shipment (unit)
TAC_d	the annual total relevant cost of the distributor

Retailer's parameters

A_r	the retailer ordering cost per contract
h_r	stock holding cost per unit per year ($/unit/year)
τ_r	the fixed transportation cost of receiving a shipment from distributor ($/shipment)
Q_r	shipment size from distributor to retailer in each shipment (unit)
S_r	unit selling price per item of good quality
m	number of shipment per order from distributor to retailer $m \geq 1$
w	minimum order quantity at which the trade credit is permitted
M	retailer's trade credit period offered by distributor linked to order quantity w
I_e	interest earned per dollar per year
I_p	interest payable per dollar per year

$I_r(t)$ the inventory level that changes with time t during the period T_4

T_4 the replenishment time interval and $T_4 = T_3 / \text{m} = \text{T}/\text{mn}$

TAC_r the annual total relevant cost of the retailer

3. MODEL FORMULATION

In order not to allow any shortage, the production rate P is assumed to be higher than the time dependent demand rate of the product through the parameter k (k>1, P=kD). Given that in each ordering cycle, the manufacturer delivers n shipments to the distributor and each shipment having Q_d units of products; the manufacturer uses a policy of producing nQ_d units with time dependent production rate in time T_1 shown in Fig. 1(a) and 1(b).

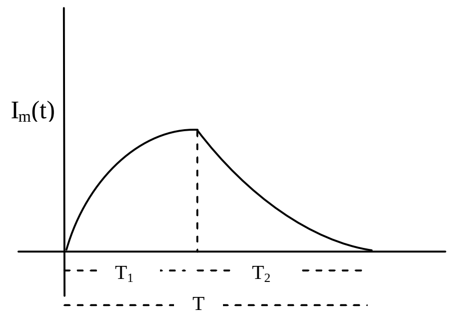

Fig. 1 (a) Manufacturer inventory level with respect to time

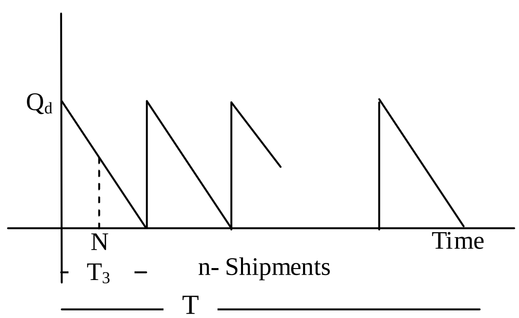

Fig. 1(b), for Distributor

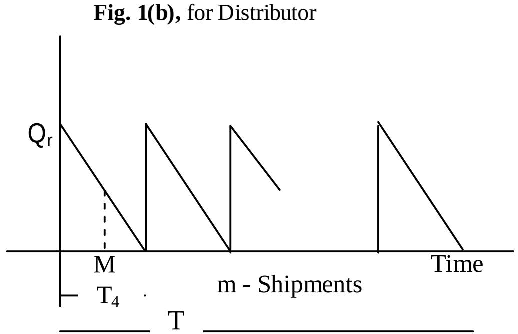

Fig. 1(c), for Retailers

Distributor again splits the quantity Q_d into m shipments and delivers Q_r units of the products to the m retailers in each shipment. So the inventory of the distribution centre resembles a step function, each step having the height of quantity Q_r (Q_d/m) shown in Fig. 1(c).

3.1 Manufacturer's model

A variable production rate starts at t=0 and continuous up to t= T_1 where inventory level reaches the maximum level. Production then stops at t= T_1 and the inventory gradually depletes to zero at the end of cycle time t=T due to consumption as shown in Fig. 1(a). Therefore, during the time interval (0 T_1), the system is subject to the effect of production and demand and the time interval (0 T_2), the system is subject to the effect of demand only. Then the change in inventory level can be described by the following differential equation

$$\frac{dI_{m1}(t)}{dt} = (k-1)(a+bt) \qquad \text{where} \quad 0 \le t \le T_1$$

And $$\frac{dI_{m2}(t)}{dt} = -(a+bt) \qquad \text{where} \quad 0 \le t \le T_2$$

with conditions $I_{m1}(0) = 0$ and $I_{m2}(T_2) = 0$ Solutions of above eq. are

$$I_{m1}(t) = (k-1)\left(at + \frac{1}{2}bt^2\right) \qquad \text{where} \quad 0 \le t \le T_1$$

and $$I_{m2}(t) = a(T_2 - t) + \frac{1}{2}b\left(T_2^2 - t^2\right) \text{ where } \quad 0 \le t \le T_2$$

In addition, from the boundary condition $I_{m1}(T_1) = I_{m2}(0)$, we can derive the following equation;

$$(k-1)\left(aT_1 + \frac{1}{2}bT_1^2\right) = aT_2 + \frac{1}{2}bT_2^2 \qquad \text{....... (1)}$$

The individual costs are now evaluated before they are grouped together

1. Annual set-up cost (SC_m) = A_m/T
2. Annual transportation cost (TC_m) = τ_m.n/T
3. Annual stockholding cost (HC_m)

$$= \frac{h_m}{T}\left[\int_0^{T_1} I_{m1}(t)\,dt + \int_0^{T2} I_{m2}(t)\,dt\right]$$

$$= \frac{h_m}{T}\left[(k-1)\left(\frac{1}{2}aT_1^2 + \frac{1}{6}bT_1^3\right) + \left(\frac{1}{2}aT_2^2 + \frac{1}{6}bT_2^3\right)\right]$$

4. Opportunity interest loss per unit time in n shipments

$$\text{IL}_m = \frac{I_m.S_m.n}{T}\int_0^N D(t)\,dt = \frac{I_m.S_m.n}{T}\left(aN + \frac{1}{2}bN^2\right)$$

The annual total relevant cost of the manufacturer

$$TAC_m = SC_m + TC_m + HC_m + IL_m \qquad \text{………. (2)}$$

Determination of the values of T_1 and T_2

In this section, we shall determine the values of T_1 and T_2

Solving eq. (1) and $T_1 + T_2$ =T, we find

$$T_1 = \frac{-(bT+ka) + \sqrt{(k-1)b^2T^2 + 4(k-1)abT + k^2a^2}}{(k-2)b} \qquad \text{………. (3)}$$

$$T_2 = \frac{\left[(k-1)bT + ka\right] + \sqrt{(k-1)b^2T^2 + 4(k-1)abT + k^2a^2}}{(k-2)b}$$

Where k >2 ……… (4)

Because if k=1 that is, production and demand rate are same, there is no accumulation and stock will finish at the end of T_1. It means T_2 = 0. If k=2, the values of T_1 and T_2 are undetermined (0/0 form), that is two time production rate of the demand rate is not possible in this situation with this rate (a+bt). Therefore all discussion will be done for the value of k>2.

Lemma 1. For positive value of T_1 and T_2

Let us suppose that $T_1 > 0$ then

$$\frac{-(bT+ka)+\sqrt{(k-1)b^2T^2+4(k-1)abT+k^2a^2}}{(k-2)b}>0$$

$$\sqrt{(k-1)b^2T^2+4(k-1)abT+k^2a^2}>(bT+ka)$$

$$(k-1)b^2T^2+4(k-1)abT+k^2a^2>b^2T^2+k^2a^2+2kabT$$

$(k-2)b^2T^2+2(k-2)abT>0$ [k > 2, so (k-2) is a positive number]

$bT(bT+2a)>0$ [If l.m > 0, then either both positive or both negative]

Here according to the assumptions a, b, k and T all are positive, then

$(bT+2a)>0$ is true. Therefore T_1 is a positive number.

And same way we can show that T_2 is also a positive number.

3.2 Distributor's model

The level of inventory $I_d(t)$ gradually decreases to meet demands to retailers. It is shown in Fig.1(b). Hence the variation of inventory with respect to time t can be described by the following differential equations;

$\frac{dI_d(t)}{dt}=-(a+bt)$ where $0\le t\le T_3$ and $I_d(T_3)=0$,

consequently solution is given by

$I_d(t)=a(T_3-t)+\frac{1}{2}b(T_3^2-t^2)$ where $0\le t\le T_3$ and $T_3=T/n$ and the order quantity is

$$Q_d=I_d(0)=aT_3+\frac{1}{2}bT_3^2 \qquad \text{...... (5)}$$

The individual costs are now evaluated before they are grouped together

1. Annual ordering cost (OC_d) = nA_d/T
2. Annual stockholding cost (excluding interest charges)

   $$HC_d=\frac{n.h_d}{T}\int_0^{T_3}I_d(t)dt=\frac{n.h_d}{T}\left(\frac{1}{2}aT_3^2+\frac{1}{6}bT_3^3\right)$$

3. The distributor incurs two annual shipment cost element, one for receiving shipments from manufacturer and the other for delivering shipments to the retailers

   The shipment cost for receiving (TC_{d1}) = $\tau_{d1}.n/T$
   The shipment cost for delivering (TC_{d2}) = $\tau_{d2}.mn/T$

4. Opportunity interest loss per unit time in mn shipments

   $$IL_d=\frac{I_o.S_d.\text{mn}}{T}\int_0^M D(t)dt=\frac{I_o.S_d.\text{mn}}{T}\left(aM+\frac{1}{2}bM^2\right)$$

5. Regarding interest earned and payable, we have following two possible cases based on the value of T_3 and N

Case-I when $N\le T_3$

i. Interest earned per year in n shipments

$$IE_{d1}=\frac{n.I_e.S_d}{T}\int_0^{T_3}(T_3-t)D(t)dt=\frac{n.I_e.S_d}{T}\left(\frac{1}{2}aT_3^2+\frac{1}{6}bT_3^3\right)$$

ii. Interest payable per year in n shipments

$$IP_{d1}=\frac{n.I_p.S_m}{T}\int_N^{T_3}I_d(t)dt$$
$$=\frac{n.I_p.S_m}{T}\left[\frac{1}{2}a(T_3-N)^2+\frac{1}{6}b\left(2T_3^3-3T_3^2N+N^3\right)\right]$$

Case-II when $N\ge T_3$

i. Interest earned per year in n shipments

$$IE_{d2}=\frac{n.I_e.S_d}{T}\left[\int_0^{T_3}(T_3-t)D(t)dt+(N-T_3)\int_0^{T_3}D(t)dt\right]$$
$$=\frac{n.I_e.S_d}{T}\left[N\left(aT_3+\frac{1}{2}bT_3^2\right)-\left(\frac{1}{2}aT_3^2+\frac{1}{3}bT_3^3\right)\right]$$

In this case, no interest charges are paid for the items kept in stock, i.e. $IP_{d2}=0$

Therefore, the annual total relevant cost of the distributor is

$$TAC_d=\begin{cases}TAC_{d1} & if \quad N\le T_3\\ TAC_{d2} & if \quad N\ge T_3\end{cases} \quad \text{where} \qquad \text{……….. (6)}$$

$TAC_{d1} = OC_d + HC_d + TC_{d1} + TC_{d2} + IL_d + IP_{d1} – IE_{d1}$ … (7)

$TAC_{d2} = OC_d + HC_d + TC_{d1} + TC_{d2} + IL_d + IP_{d2} – IE_{d2}$ … (8)

3.3 Retailer's model

The level of inventory $I_r(t)$ gradually decreases to meet demands to customers. It is shown in Fig.1(c). Hence the variation of inventory with respect to time t can be described by the following differential equations

$\frac{dI_r(t)}{dt}=-(a+bt)$ where $0\le t\le T_4$ and $I_r(T_4)=0$,

consequently solution is given by

$I_r(t)=a(T_4-t)+\frac{1}{2}b(T_4^2-t^2)$ where $0\le t\le T_4$, $T_4=T_3/\text{m}$ and $T_4=T/\text{m}n$ and the order quantity is

$$Q_r=I_r(0)=aT_4+\frac{1}{2}bT_4^2 \qquad \text{……… (9)}$$

The individual costs are now evaluated before they are grouped together

1. Annual ordering cost (OC_r) = mnA_r/T
2. Annual stockholding cost (excluding interest charges)

   $$HC_r=\frac{mn.h_r}{T}\int_0^{T_4}I_r(t)dt=\frac{mn.h_r}{T}\left(\frac{1}{2}aT_4^2+\frac{1}{6}bT_4^3\right)$$

3. The transportation cost for receiving shipments from distributor $TC_r = \tau_r\, mn / T$
4. Regarding interest earned and payable, we have following three cases based on the values of Q_r, w, M and T_4.

Case-1 when $Q_r < w$

According to assumption, the trade credit is not permitted. The retailer must pay $S_d.Q_r$ when the order is received. Therefore,

(i) In this case, no earned interest, i. e., $IE_{r1}=0$

(ii) Interest payable for the items kept in stock per year in mn shipments

$$IP_{r1} = \frac{mn.I_p.S_d}{T}\int_0^{T_4} t.D(t)\,dt = \frac{mn.I_p.S_d}{T}\left(\frac{1}{2}aT_4^2 + \frac{1}{3}bT_4^3\right)$$

Case-2 when $Q_r > w$, the fixed credit period M is permitted. Therefore two cases arise

Case-2.1 when $M < T_4$

(i) Interest earned per year in mn shipments

$$IE_{r2} = \frac{mn.I_e.S_r}{T}\int_0^{T_4} t.D(t)\,dt = \frac{mn.I_p.S_r}{T}\left(\frac{1}{2}aT_4^2 + \frac{1}{3}bT_4^3\right)$$

(ii) Interest payable per year in mn shipments

$$IP_{r2} = \frac{mn.I_p.S_d}{T}\int_M^{T_4} t.D(t)\,dt = \frac{mn.I_p.S_d}{T}\left(\frac{1}{2}a\left(T_4^2 - M^2\right) + \frac{1}{3}b\left(T_4^3 - M^3\right)\right)$$

Case-2.2 when $M \geq T_4$

(i) Interest earned per year in mn shipments

$$IE_{r3} = \frac{mn.I_e.S_r}{T}\left[\int_0^{T_4}(T_4 - t).D(t)\,dt + (M - T_4)\int_0^{T_4} D(t)\,dt\right] = \frac{mn.I_e.S_r}{T}\left[M\left(aT_4 + \frac{1}{2}bT_4^2\right) - \left(\frac{1}{2}aT_4^2 + \frac{1}{3}bT_4^3\right)\right]$$

(ii) In this case, no interest charges are paid for the items kept in stock. i. e. $IP_{r3} = 0$

Therefore, the annual total relevant cost of the retailers is

$$TAC_r = \begin{cases} TAC_{r1} & if \quad Q_r < w \\ & whenQ_r > w \\ TAC_{r2} & if \quad M < T_4 \\ TAC_{r3} & if \quad M \geq T_4 \end{cases} \quad \text{Where} \qquad \text{……….} (10)$$

$TAC_{r1} = OC_r + HC_r + TC_r + IP_{r1} - IE_{r1}$ ………. (11)

$TAC_{r2} = OC_r + HC_r + TC_r + IP_{r2} - IE_{r2}$ ………. (12)

And

$TAC_{r3} = OC_r + HC_r + TC_r + IP_{r3} - IE_{r3}$ ………. (13)

Finally, the annual total cost of the entire supply chain *TCS* is composed of the manufacturer's annual cost TAC_m, the distributor's annual cost TAC_d and retailer's annual cost TAC_r. It is important to note that having cycle time T and permissible delay periods N and M incur different annual cost to the distributor and retailer. Hence, the annual total relevant cost of the entire system will also be different for different cases

Case-I when $N \leq T_3$

The annual total cost of the system can be written as

$$TCS^{\alpha} = \begin{cases} TCS_1 & if \quad Q_r < w & (a) \\ & whenQ_r > w \\ TCS_2 & if \quad M < T_4 & (b) \\ TCS_3 & if \quad M \geq T_4 & (c) \end{cases}$$

……… (14)

Where

$TCS_1 = TAC_m + TAC_{d1} + TAC_{r1}$

$TCS_2 = TAC_m + TAC_{d1} + TAC_{r2}$ and

$TCS_3 = TAC_m + TAC_{d1} + TAC_{r3}$

Case-II when $N \geq T_3$

$$TCS^{\beta} = \begin{cases} TCS_4 & if \quad Q_r < w & (a) \\ & whenQ_r > w \\ TCS_5 & if \quad M < T_4 & (b) \\ TCS_6 & if \quad M \geq T_4 & (c) \end{cases}$$

……… (15)

Where

$TCS_4 = TAC_m + TAC_{d2} + TAC_{r1}$,

$TCS_5 = TAC_m + TAC_{d2} + TAC_{r2}$ and

$TCS_6 = TAC_m + TAC_{d2} + TAC_{r3}$

This study develops an integrated production-inventory model with a certain permissible delay in payment for distributor and retailers. An approximate models with a single manufacturer a single distributor and a single retailer is developed to derive the optimal production policy and lot size. Since T_4=T/mn, T_3=T/n and the values of T_1 and T_2 are from eq. (3) and (4), the problem can stated as an optimization problem and it can be formulated as

Minimize: TCS (m, n, T) = $TAC_m + TAC_d + TAC_r$ .... (16)

Subject: 0 ≤T, 0≤m, 0≤n .... (17)

4. SOLUTION PROCEDURE

The optimization technique is used to minimize (16) to derive T as follow;

Step1. Since the number of delivery per order m and n are an integer value, start by choosing an integer value of m, $n \geq 1$

Step2. Take the derivative of TCS (m, n, T) with respect to T and equate the result to zero.

$TCS^{I}(\text{m}, \text{n}, T) = 0$ and solving for T

Step3. Find those values of T from step 2 for that $TCS^{II}(\text{m}, \text{n}, T) > 0$

Step4. Using these values of T in eq. (16) and find the minimum value of TCS

Step5. Repeat steps 2 and 3 for all possible values of m, n until the minimum TCS *(m^*, n^*, T^*)* is found. The TCS *(m^*, n^*, T^*)* values constitute the optimal solution that satisfy the condition mentioned in step 3.

Step6. Derive the T_1^*, T_2^*, T_3^*, T_4^*, Q_m^*, Q_d^*, Q_r^*, TAC_m^*, TAC_d^*, TAC_r^* .

5. NUMERICAL EXAMPLES

Optimal production and replenishment policy to minimize the total system cost may be obtained by using the methodology proposed in the proceeding section. The following numerical examples are illustrated the model. The values of parameters adopted in this study are $A_m = 500$, $A_d = 300$, $A_r = 100$, $\tau_m = 300$, $\tau_{d1} = 70$, $\tau_{d2} = 150$, $\tau_r = 50$, $I_e = 0.2$, $I_p = 0.3$, $I_m = 0.1$, $I_o = 0.15$, $s_m = 8$, $s_d = 10$, $s_r = 12$, $h_m = 2$, $h_d = 3$, $h_r = 5$, $a = 10$, $b = 5$, $k = 3$, $M = 2$, $N = 3$. The computational results are shown below as

For $n = 2,\ m = 3\ and\ w = 25$

Using eq. 14 (a) we found the value of $T = 9.02$ then $T_1 = 3.6,\quad T_2 = 5.4,\quad T_3 = 4.5\quad T_4 = 1.5,$ $Q_r = 20$ and the optimal value of the total system cost $TCS = \$659$. We can see that the results we have found from this analysis satisfied conditions $N \leq T_3\ and\ Q_r < w$, i. e. There is no permissible delay to the retailer.

For $n = 1,\ m = 2\ and\ w = 25$

Using eq. 14 (b) we found the value of $T = 4.70$ then $T_1 = 1.88,\quad T_2 = 2.82,\ T_3 = 4.70\ T_4 = 2.35,$ $Q_r = 37$ and the optimal value of the total system cost $TCS = \$565$. We can see that the results we have found from this analysis satisfied conditions $Q_r > w,\ N \leq T_3\ and\ M \leq T_4$

For $n = 1,\ m = 3\ and\ w = 25$

Using eq. 14 (c) we found the value of $T = 5.40$ then $T_1 = 2.16,\quad T_2 = 3.24,\ T_3 = 5.40\ T_4 = 1.8,$ $Q_r = 28$ and the optimal value of the total system cost $TCS = \$586$. We can see that the results we have found from this analysis satisfied conditions $Q_r > w,\ N \leq T_3\ and\ M \geq T_4$

For $n = 3,\ m = 1\ and\ w = 30$

Using eq. 15 (a) we found the value of $T = 7.7$ then $T_1 = 3.08,\quad T_2 = 4.62,\ T_3 = 2.8\ T_4 = 2.8,$ $Q_r = 25$ and the optimal value of the total system cost $TCS = \$712$. We can see that the results we have found from this analysis satisfied conditions $N \geq T_3\ and\ Q_r < w$, i. e. There is no permissible delay to the retailer.

For $n = 4,\ m = 2\ and\ w = 30$

Using eq. 15 (b) we found the value of $T = 11.6$ then $T_1 = 4.64,\quad T_2 = 6.96,\ T_3 = 2.9\ T_4 = 1.45,$ $Q_r = 40$ and the optimal value of the total system cost $TCS = \$682$. We can see that the results we have found from this analysis satisfied conditions $Q_r > w,\ N \leq T_3\ and\ M \leq T_4$

For $n = 4,\ m = 1\ and\ w = 30$

Using eq. 15 (c) we found the value of $T = 10.22$ then $T_1 = 4.08,\quad T_2 = 6.13,\ T_3 = 2.5\ T_4 = 2.5,$ $Q_r = 42$ and the optimal value of the total system cost $TCS = \$664$. We can see that the results we have found from this analysis satisfied conditions $Q_r > w,\ N \leq T_3\ and\ M \geq T_4$

6. CONCLUSION

Using various methods to reduce costs has become the major focus for supply chain management. In order to decrease the joint total cost, the manufacturer, distributor and retailer are willing to invest in reducing the different costs. This paper develops the integrated manufacturer-distributor-retailer models with different type of permissible delays in payments (manufacturer offers distributor and distributor provides order linked trade credit to the retailer) to determine the optimal replenishment time interval and replenishment frequency to reduce the total system costs to the all. We have provided an example, discussed all the cases and found the results that satisfied all the required conditions. We have used some realistic costs like transportation cost and cost for opportunity interest loss due to permissible delay. The proposed model can be extended in several ways. For example, we may generalize the model to allow for imperfect production process and deteriorating items.

7. REFERENCES

[1] Abad, P. L. and Jaggi, C.K. 2003. Joint approach for setting unit price and the length of the credit period for a seller when end demand is price sensitive. International Journal of Production Economics 83, 115–122.

[2] Chung, K.J., 1998. A theorem on the determination of economic order quantity under conditions of permissible delay in payments. Computers and Operations Research, 25, 49-52.

[3] Chang, H. C., Ho, C. H., Ouyang, L.Y. and Su, C. H., 2009. The optimal pricing and ordering policy for an integrated inventory model when trade credit linked to order quantity. Applied Mathematical Modelling, 33, 2978–2991.

[4] Chang, C.T., Ouyang, L.Y., Teng, J.T. and Cheng, M.C. 2010. Optimal ordering policies for deteriorating items using a discounted cash-flow analysis when a trade credit is linked to order quantity. Computers and Industrial Engineering, 59, 770–777.

[5] Chen, J. and Bell, P.C. 2011. Coordinating a decentralized supply chain with customer returns and price-dependent stochastic demand using a buyback policy. European Journal of Operational Research, 212 (2), 293–300

[6] Chung, K. J. and Cardenas-Barron, 2013. The simplified solution procedure for deteriorating items under stock dependent demand and two level trade credits in the supply chain management. Appl. Math. Model, 37, 4653-4660.

[7] Goyal, S. K. 1985. Economic order quantity under conditions of permissible delay in payments. Journal of the Operational Research Society 36, 335-338.

[8] Goyal, S. K., Teng, J.T. and Chang, C.T. 2007. Optimal ordering policies when the supplier provides a progressive interest-payable scheme. European Journal of Operational Research, 179,404–413

[9] Haq, A. N., Vrat, P., and Kanda, A. 1991. An integrated production-inventory-distribution model for manufacture of urea: a case. International Journal of Production Economic, 39, 39–49.

[10] Ishii, K., Takahashi, K., and Muramatsu, R. 1988. 'Integrated production, inventory and distribution systems. International Journal of Production Research *26*(3), 473–482

[11] Jaggi, C.K., Goyal, S.K. and Goel, S.K. 2008. Retailer's optimal replenishment decisions with credit-linked demand under permissible delay in payments. European Journal of Operations Research, 190, 130-135.

[12] Jaber, M.Y. and Goyal, S.K. 2008. Coordinating a three-level supply chain with multiple suppliers, a vendor and multiple buyers. Int. J. Production Economics 116, 95-103

[13] Khouja, M., and Mehrez, A. 1996. Optimal inventory policy under different supplier credit policies. Journal of Manufacturing Systems, 15, 334-339

[14] Liao, J.J. 2008. An EOQ model with non instantaneous receipt and exponentially deteriorating items under two-level trade credit. International Journal of Production Economics, 113,852–861

[15] Ouyang, L. Y., Wu, K. S. and Yang, C. T. 2006. A study on an inventory model for non instantaneous deteriorating items with permissible delay in payments', Computers & Industrial Engineering, 51, 637-651.

[16] Omar, M., Sarker, R. and Othman, W. A. M. 2013. A just-in-time three-level integrated manufacturing system for linearly time-varying demand process. Applied Mathematical Modelling, 37, 1275-1281.

[17] Rau, H. Wu, M.Y. and Wee, H.M. 2003. Integrated inventory model for deteriorating items under a multi-echelon supply chain environment. International Journal of Production Economics, 86, 155–168

[18] Singh, S. R., Singh, T. J. & Singh, C. 2007. Perishable inventory model with quadratic demand, partial backlogging and permissible delay in payments. International review of Pure and Applied Mathematics, 3 (2), 199-212.

[19] Singh, N., Vaish, B. & Singh, S. R. 2010. An EOQ model with Pareto distribution for deterioration, Trapezoidal type demand and backlogging under trade credit policy. The IUP Journal of Computational Mathematics, 3 (4), 30-53.

[20] Singh, S. R., Kumar, N. & Kumari, R. 2011. Two warehouse fuzzy inventory model under the condition of permissible delay in payments. International Journal of Operational Research, 11 (1), 78-99.

[21] Su, C.H. 2012. Optimal replenishment policy for an integrated inventory system with defective items and allowable shortage under trade credit. International Journal of Production Economics, 139, 247–256.

[22] Singh, C. & Singh, S. R. 2012. Integrated supply chain model for perishable items with trade credit policy under imprecise environment. International Journal of Computer Applications, 48 (20), 41-45.

[23] Soni, H. N. 2013. Optimal replenishment policies for deteriorating items with stock sensitive demand under two-level trade credit and limited capacity. Appl. Math. Model, 37, 5887-5895

[24] Thangam, A. and Uthaykumar, R. 2009. Two echelon trade credits financing for perishable items in a supply chain when demands on both selling price and credit period' Computers & Industrial Engineering, 57, 773-786

[25] Woo,Y.Y., Hsu, S.L. and Wu, S. 2001. An integrated inventory model for single vendor and multiple buyers with ordering cost reduction. International Journal of Production Economics, 73, 203–215.

[26] Yadav, D.,Singh, S. R. & Kumari, R. 2013. Retailer's optimal policy under inflation in fuzzy environment with trade credit. International Journal of Systems Science, DOI:10.1080/00207721.2013.801094