

Protocol Built by Chaotic Map and Elliptic Curve Cryptography for Key Exchange

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Abstract: The elliptical curve system has received great interest in the science of security systems, and this has a great number of advantages. When linked with chaotic systems, it gave a broad comprehensiveness in the science of finding common keys between the two parties or in a system as a server that distributes the common keys among the participants in the distress.

The proposed algorithm reduces processor load, reduces power consumption, increases processing speed, enhances storage efficiency, requires smaller certificates, and is good at saving bandwidth. Where ECC gives high-level arithmetic operations ECC is an algebraic structure in wide areas for a large number of points and any point in the Cartesian coordinates, as well as for a number of prime numbers generated from the chaotic three-dimensional system using the multiplication and addition algorithm of the elliptical curved system. This paper suggested a new event in the implementation method: the inputs to the elements of the controlling parameters and the initial conditions of the symmetric chaotic functions produce an ECC by a secret shared key between two parties or a number of parties participating in the group, and all these points of the share keys lie on the points of the curve. and this key technology is available for authentication, confidentiality, and non-repudiation.

Keywords: Diffie-Hellman; communication channel; ECC; chaotic map; share key.

1. INTRODUCTION

The main benefit of using ECC technology is that it provides the same level of protection with a smaller switch length compared to RSA. Thus, it will reduce the execution time of the algorithm processing as well as reduce the processing cost [1, 2]. Elliptic Curve Cryptography (ECC) is a popular algorithm that provides a higher level of security.

Smaller key sizes with lower resource consumption, which makes it an ideal choice for resource-constrained devices. The unpredictability introduced by the chaotic maps in the proposed algorithm is in addition to the fact that the algorithms make the model easy to implement, fast, and powerful against the attacker. The more coordinated the chaotic functions, the higher the level of safety and the better the quality. Chen et al. [3] suggest a symmetric image encoding algorithm with a 3D cat map and logistics map for mixing image data with an extra layer of confusion between them, resulting in an image with normal encoding after every two rounds of mixing. The cryptography also uses Chen's chaotic key generation system. Parrick and others [4] used as a result of two logistics maps to choose one of the eight randomly designed modes to encode every pixel in the image. Designed by Liu and Wang [5] as a one-time master-stream encoding for color image encoding on a piecewise linear chaos map (PWLCM) to calculate mainstream color image coding. The main stream is based on another sequence obtained from the Chebyshev map.

The authors, Moncef Amara and Amar Siad [6], discussed elliptical curve coding (ECC) and its performance in terms of speed of implementation and security compared to traditional encryption algorithms like RSA. Points are generated by specifying the prime number P . The authors Ali Soleyamani, Md. Jan Nordin, and Zulkarnain Md. Ali discussed [7]. ECC The map table method is used to encode the image. ECC points are generated by choosing the largest Prime No. A mapping table is generated using points created

by an elliptical curve. The points generated are set image pixels to perform encoding.

Requires good ECC technology awareness and a mathematical background compared to elliptical curves. elliptic curves In addition, elliptic curves are not ellipses [8, 9], and after all, they are It is defined as an ellipse because EC is a derivative of cubic equations (1). Sangook Moon: To perform numerical operations, develop a more efficient and novel approach to scalar point multiplication from the method of double multiplication and existing addition, then apply redundant recoding that originates from the root. 4 Booth's algorithm [10] L. Young. sign to Aydos et al protocol is not safe for humans in midfield attack by any attacker [11].

Koblitz and Miller were the first to use the elliptic curve cryptography technique [12]. Ravi Kodali and N. Sarma use Elliptic Curve Cryptography symmetric encryption with the Koblitz's encoding to encode or map the data into points locating on the Elliptic Curve, and it is one of the main basics of the Elliptic Curve Cryptography [13]. J. Nafeesa Begum and others Improved multi-level defensive messaging system ECC access control defense message system Redirects a message to parties or recipients based on message criteria for quick action [14]. Several algorithms have been developed to solve the problem. However, the effectiveness of any algorithm is determined by the type of curve and the properties of k , where k is a random number [15]. Arezou and others [16] have made effective use of the elliptic curve cipher in building a three-factor authentication.

system for satellite communications when it comes to implementing ECC cloud computing security.

Kumar & Grover's and other [17] focused on applying the ECC algorithm for encryption and decryption processes to improve cloud computing security and protect privacy. Anand and Perumal [18] introduced a method to prevent any user from gaining unwanted access to confidential data stored in the cloud. (ECC) is known to be superior to public-key cryptography approaches in wireless devices. Helps reduce

device processing time. Wang and others [19] presented a work based on the concept of ECC and introduced a new way to secure ECC output. Shenet and others [20] The Java language facilitated the usage of ECC by analyzing its capabilities and dealing with digital signatures, key exchange, and key generation.

2. RESEARCH PROBLEM

The problem of generating share keys between the two parties using the Diffie-Hellman algorithm is not solved in large numbers due to the time delay in implementing the protocol with a large number of iterations, which leads to a decrease in the efficiency of the implementation of the algorithm. The research problem was solved by making the protocol algorithm strong, solid, and robust against fraudulent party attacks. This is done by using chaotic functions that are related to certain properties, such as the sensitivity of parameters and initial conditions[21,22,23].

The research problem is solved by making the protocol algorithm robust against fraudulent party attacks, and this is done by using chaotic functions associated with certain properties such as sensitivity parameters and initial conditions. And the results of that algorithm are from research [24,25,26]. It led to the generation of common keys between the two parties or between the two parties for the common keys, and these shared keys are in the form of points with arranged pairs and are located exclusively on the points of the curve, which makes the protocol algorithm strong and strong against fraudulent attacks.

3. ELLIPTIC CURVES OVER REAL NUMBERS

Elliptic curves are not elliptical[12]. It is so named because it is described by cubic equations, similar to those used to calculate the circumference of an ellipse. In general, cubic equations for elliptical curves take the following form, known as the Weierstrass equation (1):

$$y^2+axy+by=x^3+cx^2+dx+e.....(1)$$

where a,b,c,d and e are real numbers and take on values in the four real numbers. For our purpose, it is sufficient to limit ourselves to the form of equations (2).

$$y^2=x^3+ax+b.....(2)$$

Such equations are said to be cubic, or of three degrees, because the highest exponent they contain is a three [21, 22]. The elliptic curve (E) is a non-singular algebraic plane curve defined over a finite field F_p , where $x, y, a, b \in F_p$. The clarity of this equation (2) is shown in figure 1

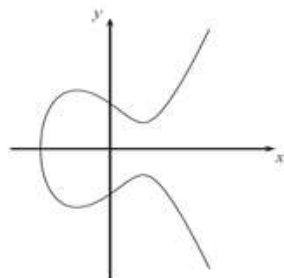


FIGURE 1. An Elliptic Curve E defined over a finite field F_p

3.1. Addition And Multiplication Of Elliptic Points

Assuming P_1, P_2 and R be three points on ellipse curve[23]. Let

The addition stage first method used is $R=P_1+P_2=(x_R, y_R)$ is determined by the following rules(3) and (4).

$$m=(y_2-y_1)/(x_2-x_1) \bmod p \text{ if } P_1 \text{ not equal } P_2 \text{ or}$$

$$m=(3x_1^2+a)/(2y_1) \bmod p \text{ if } P_1 \text{ equal } P_2$$

$$x_R=(m^2-x_1-x_2) \bmod P(3)$$

$$y_R=(m(x_1-x_R)-y_1) \bmod P(4)$$

Where a,b are coefficients of equation (2) and x, y are coordinates.

The Multiplication stage second is defined as repeated addition; for example,

$$4P_1=P_1+P_1+P_1+P_1$$

For example, let $P_1=(3,10)$ and $P_2=(9,7)$ then

$$m=((7-10)/(9-3)) \bmod 23=(-3/6) \bmod 23=(-1/2) \bmod 23=11$$

$$x_R=(11^2-3-9) \bmod 23=17$$

$$y_R=(11(3-17)-10) \bmod 23=20$$

$$P_1+P_2=(17,20)$$

To find $2P_1$

$$m=((3(3^2)+1)/2*10) \bmod 23=6$$

$$x_R=(6^2-3-3) \bmod 23=7$$

$$y_R=(6(3-7)-10) \bmod 23=12$$

$$\text{And then } 2P_1=(7,12)$$

Note: To generate a curve with about 2^{160} points, a prime with a length of about 160 bits is required.

4. AN OVERVIEW ON HENON CHAOTIC MAP SYSTEM

In this section, an overview on Henon chaotic map system as important one of the 3-D chaotic map systems, which is used in this work. Henon chaotic map system is described by formula 5 which illustrates a set of the three function of Henon chaotic map system[24,25,26].

$$\begin{aligned} x(i+1) &= a - (y(i)^2) - b * (z(i)) & | \\ y(i+1) &= x(i) & |(5) \\ z(i+1) &= y(i) & | \end{aligned}$$

when initial values $1.54 < |a| < 2, 0 < |b| < 1$. and $-0.9 \leq (x \text{ or } y \text{ or } z) \leq 1$

$x(1)=1; y(1)=0; z(1)=0;$ %% Initial conditions The initial value are $x=1, y=0.1, z=0,$

$N=5000;$ %% let N is the number of iterates example

$a=1.6; b=0.2;$ %% Sets the parameters example it has a chaotic attractor as shown in Fig.(2).

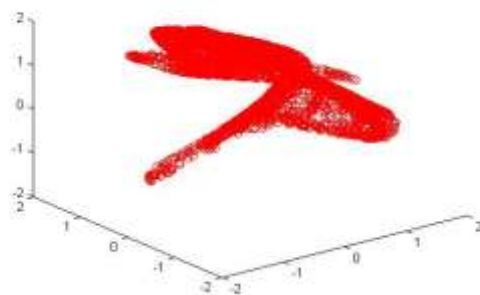


Fig.(2) Three Dimension Henon Map

5. DESIGN PROTOCOL BASED ON HENON CHAOTIC MAP AND ECC Square

Discusses the design of the proposed algorithm for the shared key exchange protocol between the parties or parties using chaotic functions of three dimensions in order to create a secure channel in order to exchange information in a reliable and secure manner against hackers and attackers of the protocol shown in the figure (3).

All the notations used in the Henon maps protocol are described in Table 1. Assume Alice and Bob are two participants in a key agreement process. The algorithm consists of the following parts:

STEP 1 :Inlet two parameter a , b and three Initial conditions X_0, Y_0, Z_0 to the Henon chaotic map system .

STEP 2: Operation between multiplication or any operation between them output henon maps.

STEP 3:Extract primary P_i from step 2.

STEP 4:Alice compute Y_a .

In the form of ordered pairs calculated from the points through the proposed algorithm and its output, which is located within the points of the elliptic curve then send to Bob.

STEP 5:Bob compute Y_b .

In the form of ordered pairs calculated from the points through the proposed algorithm and its output, which is located within the points of the elliptic curve. then send to Alice.

STEP 6 :Both Alice and Bob are compute K =share key.

In the form of ordered pairs calculated from the points through the proposed algorithm and its output, which is located within the points of the elliptic curve.

Table 1: Notations used in Henon’s Maps Protocol

Symbol	Definition
A,B	Identifiers of Alice and Bob, respectively
X_a, X_b	Private key for A and B
P	Primary number hidden
Y_a = In the form of ordered pairs calculated from the points through the proposed algorithm and its output, which is located within the points of the elliptic curve	Send to B
Y_b = In the form of ordered pairs calculated from the points through the proposed algorithm and its output, which is located within the points of the elliptic curve	Send to A
K = In the form of ordered pairs calculated from the points through the proposed algorithm and its output, which is located within the points of the elliptic curve	Finally established session key between Alice and Bob
$K=H_a=H_b$	note

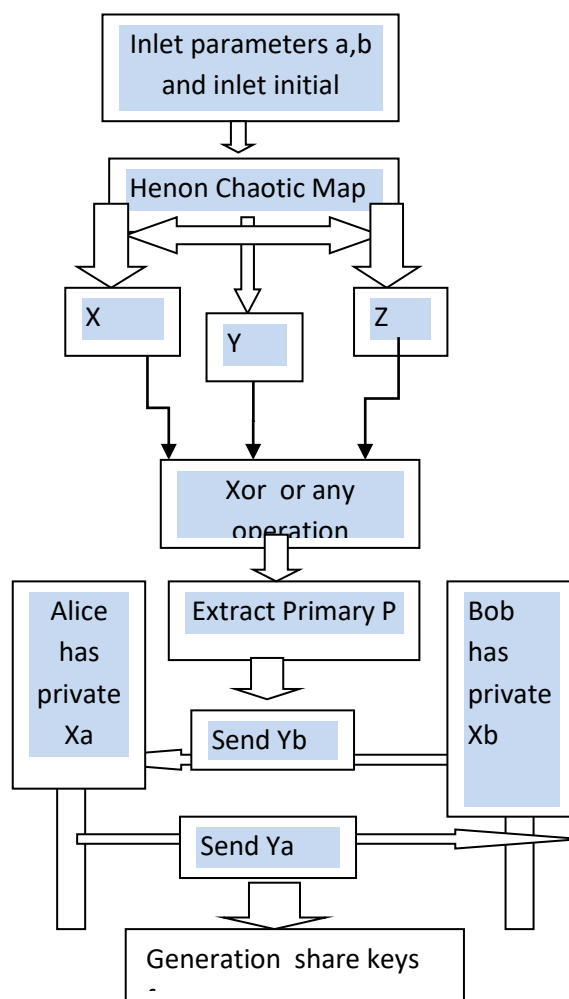


Fig. (3) General diagram of the protocol relay on chaotic and ECC

An important steps in design the proposed protocol :Extract primary P_i from step 2.
 First : Let's give a more detailed definition of a prime number.
 The algorithm that was used to design and extract the prime numbers is shown as in the table(2)
 The algorithm that was used to design and extract the prime numbers is shown as in the table(2)

Table (2) Prepare the primes numbers from chaotic sequences results

```

Algorithm Prepare the prime numbers from step 2
Stage
Process:
Input:
P1: chaotic sequences results from step 2
N=number of iteration
Output:
Prepare the prime numbers sequences
Initial:
m=0;
Loop i ← 1 to N
    if(isprime(p1(i)))
        m=m+1
        hh(m)=p1(i)
    end if
End Loop i;
Loop i ← 1 to m
    if hh(m)>=2
        print ←prime number hh(m)
    end if
End Loop m;
End.
    
```

Second : Alice will need the primes generated by the proposed algorithm, and then she will need the private key X_a to generate the public keys in the form of ordered pairs located within the points of the ellipse and send them to Bob. Likewise, Bob will also need the primes generated by the proposed algorithm, and he has the private key X_b to generate Public keys in the form of ordered pairs are located within the points of the ellipse and send to Alice.

Third: Here each party will generate the common key or a number of common keys and also be in the form of arranged pairs and located within the curve points of the ellipse, and these keys can be distributed to the rest of the participants in the exchange of information within the security communication channels.

6. ANALYZE AND EFFICIENCY OF THE PROPOSED PROTOCOL

In this paper , a practical programs of a proposed algorithm and a practical programs of all experimental and security analysis tests are designed by using MATLAB language release R2012a for 64bit Windows 7 Home Premium operating system. The computer used to perform these tests is a Dell Laptop with Intel (R) Core™ i3-3217u CPU@ 1.8GHz and 6 GB installed memory.

We have two parameter $a=1.4$ $b=0.3$ and three initial condition $X_0=0.1$, $Y_0=0.1$ and $Z_0=0.3$

Number of Iteration =50
 Alice has private key $X_a=3$
 Bob has private key $X_b=9$

Alice and Bob are Compute P_i as the hide from chaotic map three dimension

P_i =primary number =2441 281 293 1867 269 2113 139 331 1447 2063 307

Extract from each prime number a number of pairs that lie at the points of the curve of equation (1) for the following algorithm of generating a number of points for the arranged pairs that fall within the points of the ellipse in table(3).

Table(3) Algorithm of generate a number of points for the arranged pairs that fall within the points of the ellipse

```

Algorithm Prepare the generate a number of points for the arranged pairs that fall within the points of the ellipse.
Process:
Input:
P: primary number
Output:
Prepare the generate a number of points GEN( a,b,p) sequences
Initial:
i=1;
G=[];
for x=0:p
    for y=0:p
        if mod(y^2,p)==mod((x^3+a*x+b),p) % This equation must be satisfied by x and y for the given value of a, b and p.
            G(i,:)= [x y];
            i=i+1;
        else
            end;end;end; End.
    
```

The results after applying the algorithm in the table (3) are shown:

```

GEN( a, b, p)
>> [G] =GEN( 2,2, 2441)
At p=2441
G =
     0     837
     0    1604
     1     759
     1    1682
     4     131
     4    2310
     5     523
     5    1918
     6     899
     6    1542
    11     415
    
```

And so continue from the result. But we choose one of those pairs that lie within the points of the curve of equation (2).

And let the pair be in the fifth sequence, which is

```

     4    131
At p= 281
>> [G] =GEN( 2,2,281)
G =
     0    132
     0    149
     1     75
     1    206
     2     24
     2    257
     3     63
     3    218
     5     74
     5    207
     7     42
     7    239
     8    116
    
```

8 165
 11 136

Table(4) Multiplication algorithm for the ellipse curve

```

Algorithm Prepare the generate a number of points for
the arranged pairs that fall within the points of the
ellipse.
Process:
Input:
P: primary number; a,b,p,n,x,y
Output:
Prepare the results of multiple private key with function
GEN generate a number pair sequences
Initial:
if n==1
    resx = x;
    resy = y;
    return;
end
if n>=2
    [xsub,ysub]=NP(a,b,p,n-1,x,y);
    if xsub==Inf && ysub == Inf
        resx=Inf;
        resy=Inf;
    else
        [resx,resy]=Add(a,b,p,x,y,xsub,ysub);
    End;end;end;
End.
    
```

Table(5) The addition algorithm for the ellipse

```

Algorithm Prepare the generate a number of points for
the arranged pairs that fall within the points of the
ellipse.
Process:
Input:
P: primary number; a,b,p,n,x,y
Output:
Prepare the results of multiple private key with function
GEN generate a number pair sequences
Initial:
function [ resx,resy ] = Add( a,b,p,x1,y1,x2,y2 )
if x1==x2 && y1==y2
    k=modfrac(3*x1^2+a,2*y1,p);
    resx = mod(k^2-x1-x2,p);
    resy = mod(k*(x1-resx)-y1,p);
end
if x1==x2 && y1~y2
    resx = inf;
    resy = inf;
end
if x1 ~= x2
    k=modfrac(y2-y1,x2-x1,p);
    resx = mod(k^2-x1-x2,p);
    resy = mod(k*(x1-resx)-y1,p);
end;End.
    
```

And so continue from the result. But we choose one of those pairs that lie within the points of the curve of equation (2). And let the pair be in the fifth sequence, which is 2 24

The process will continue for the rest of the prime numbers generated by the chaotic three-dimensional calculation algorithm.

The next step is:

The multiplication algorithm for the ellipse curve is algorithm of NP .It is shown in Table No.(4)

Note that the private key for Alice =3. The result was the creation of a file for each party, as follows:

First:

Private key for Alice=3

And because the addition is multiplication, for example, $P+P=2P$ because p first same as p another.

But $3P=P+P+P$ we need algorithm add curve ellibts because $3P=P+P+P=2P+P$ then $2P$ not similar to P .

Say to $Q=2P$ then $3P=2P+P=Q+P$

And as follows for the addition algorithm .It is shown in Table No.(5) for the ellipse.

When we use the addition and multiplication algorithm together, we will get the following results.

Note that the private key for Alice =3

Alice compute YA1 of the Elliptic curves then send to Bob YA1=

470 485

11 136

254 233

241 1183

156 8

2064 140

53 106

295 291

961 551

932 471

158 58

And also ,Private key for Bob=9

Bob compute YB1 of the Elliptic curves then send to Alice

YB1=

288 579

224 118

166 263

1851 1044

251 66
 878 1195
 33 137
 239 142
 727 100
 1768 1808
 245 157
 Alice compute Share key of the Elliptic curves
 K=
 1708 39
 60 95
 196 228
 681 313
 134 13
 736 439
 34 68
 74 237
 50 667
 1245 1193
 300 140

Bob compute Share key
 K=
 1708 39
 60 95
 196 228
 681 313
 134 13
 736 439
 34 68
 74 237
 50 667
 1245 1193
 300 140

When Alice receives the pop file, Alice will also use the multiplication and addition algorithm to find the common keys between Alice and Bob, and also does the same process for Bob's steps, and the results are as follows for the shared keys

The table of share keys obtained by Alice from the algorithm
 1708 39
 60 95
 196 228
 681 313
 134 13

736 439
 34 68
 74 237
 50 667
 1245 1193
 300 140
 The table of share keys obtained by Bob from the algorithm
 1708 39
 60 95
 196 228
 681 313
 134 13
 736 439
 34 68
 74 237
 50 667
 1245 1193
 300 140

And because the time taken to implement is of great importance to the protocol, it depends on the number of iterations that must be implemented for the chaotic Henon-maps three-dimensional functions. The higher the number of iterations, the longer the time for executing the protocol algorithm and the results shown as shown in the table (6), which shows the relationship between the time taken to implement the algorithm with the number of iterations to implement the protocol.

Table (6) Correlation time taken to implement the algorithm with the number of iterations

Number of iteration	taken to implement the algorithm	Number of participants in group
25	6.041915	6
50	8.808280	11
60	9.243908	12
70	13.124462	14
85	13.040652	14

7. Analysis of the key space

In the proposed algorithm, Henon's chaotic function maps are generated, and each requires a control parameters value and a sequential initial conditions value for the chaotic map. where they are used as input of algorithm protocol keys, if the

underlying control variables and values are precisely in 10^{14} ,

the total space of the Key $10^5 \times 10^5 \times 10^5 \times 10^5 \times 10^5 = 10^{25}$

that length 10^{25} very good for resistant brute

8. CONCLUSIONS

The protocol for the ECC algorithm was developed using the chaotic functions of the three-dimensional Henon-Map system, the symmetry of this algorithm was introduced with the control of parameters and conditions of the initials. You get the results of this protocol algorithm to give different results while changing from the very slight input, from the implementation and analysis of the algorithm as presented, the following conclusions are obtained:

1. Prime numbers are hidden and not public. It is only visible in matlab code because it will be among the software accounts in Matlab code, sender code, and recipient code.

2. It is possible to increase the number of participants in the group by increasing the number of common keys that they exchange among themselves, and this work is inferred by increasing the number of iterations that are made for the three-dimensional chaotic functions of Henon maps. Also, the results showed that there were a limited number of participants. The number can be increased by increasing the power of the computer specifications.

3. Any change in the value of the corresponding input values between the two parties will make these keys, which act as input sensitive, sensitive to those changes in case of manipulation or attack, such as changing any key of the values entered into the algorithm from the control parameters or initial conditions, and it will be very sensitive because it will display results. Shared keys other than the previous original results between the two parties when manipulating any number of required bits and the algorithm will be efficient and effective for all anti brute force attack programs.

It should include trust and security between the group participants so that keeping the corresponding entries is guaranteed by certain entities of the protocol algorithm distribution and not just known to some unspecified party. If only two parties share the common key and there are no participants in the set, an arithmetic operation can be performed, such as adding up the points for the shared keys that are within the points of the ellipse for that set.

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