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Propagation of Partially Coherent Beams with Noncanonical Vortex Pairs in Chiral Media: Evolution of the Cross-Correlation Function

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Abstract: This study investigates the propagation characteristics and the evolution of the cross-correlation function (CCF) of partially coherent beams embedded with noncanonical vortex pairs through chiral media. A theoretical model is developed to describe the beam's behavior, incorporating the effects of chiral parameters, propagation distance, and noncanonical strength. Numerical simulations reveal that the handedness of the chiral medium—left circularly polarized (LCP) or right circularly polarized (RCP)—significantly influences the CCF structure. In LCP media, the number of CCF rings consistently equals the sum of the topological charges, while RCP media induce complex structural transformations, including ring separation and reconstruction. Additionally, both propagation distance and chirality parameter play crucial roles in the morphological development of the CCF. The noncanonical strength further modulates the coherence structure, facilitating transitions from nested elliptical configurations to well-separated ring patterns. These findings enhance the understanding of singular optics in complex media and offer potential applications in optical communication, sensing, and manipulation.

Keywords: Partially coherent beams; noncanonical vortices; chiral media; coherence structure; cross-correlation function

1. INTRODUCTION

Optical vortices, characterized by phase singularities and helical wavefronts, have garnered significant attention in modern optics due to their unique properties and wide applications in optical manipulation, communication, and imaging [1–3]. The topological charge (TC) of an optical vortex defines the number of 2π phase cycles around the singularity, which plays a crucial role in determining the orbital angular momentum (OAM) carried by the beam [4, 5]. Traditionally, canonical vortices with symmetric phase profiles have been extensively studied. However, noncanonical vortices—characterized by asymmetric phase distributions and non-unitary noncanonical strengths—offer additional degrees of freedom for beam shaping and coherence engineering [6, 7].

Partially coherent beams, which exhibit both spatial and temporal coherence properties, provide a powerful platform for studying the interplay between coherence and phase singularities [8, 9]. The incorporation of optical vortices into partially coherent beams has led to the emergence of novel phenomena such as self-reconstruction [10], anomalous rotation [11], and coherence singularities [12]. These behaviors are particularly pronounced when such beams propagate through complex media, including anisotropic, turbulent, or chiral environments [13–15].

Chiral media, which exhibit different refractive indices for left-handed and right-handed circularly polarized waves, introduce optical activity that rotates the polarization plane of transmitted light [16]. This property has been exploited in polarization optics, metamaterials, and biosensing [17]. However, the propagation of partially coherent beams carrying noncanonical vortex pairs through chiral media remains largely unexplored. Understanding the evolution of the cross-correlation function (CCF) under such conditions is essential for harnessing these beams in advanced optical systems.

In this paper, we theoretically and numerically investigate the propagation behavior of partially coherent beams embedded with noncanonical vortex pairs in chiral media. We derive the generalized Huygens–Fresnel integral for the cross-spectral density (CSD) function and analyze the influence of chirality, noncanonical strength, and propagation distance on the CCF. Our results reveal distinctive structural transformations in the CCF depending on the handedness of the chiral medium, offering new insights into the control of coherence and phase properties in structured light.

2. THEORETICAL MODEL

In the polar coordinate system, the electric field of the noncanonical optical vortex pairs in a scalar beam at the source plane (z = 0) can be expressed as

$$E(\mathbf{r},0) = F(\mathbf{r})G\left(\mathbf{r}\right)\exp\left(i\zeta\right) \quad (1)$$

where $\mathbf{r} = (x, y)$ is the position vector in the source plane (z = 0), ζ is an arbitrary phase, $G(\mathbf{r})$ is a Gaussian profile beam with a waist width of w_0, and $F(\mathbf{r})$ is the noncanonical vortex pairs function described by

$$F(\mathbf{r}) = F(x,y) = \left[(x-d) + isgn(l_1)Qy\right]^{|l_1|} \times \left[(x+d) + isgn(l_2)Qy\right]^{|l_2|} \quad (2)$$

where l_1 and l_2 are the topological charges of the optical vortex, d is the off-axis distance of the noncanonical vortex pairs in the x-direction, and sgn is the sign function. The complex parameter Q in Eq. (2) denotes the noncanonical factor of the optical vortex, and $Q = \pm 1$ denotes the canonical or symmetric vortex. Assuming that the statistical distribution of any phase ζ corresponds to the correlation of the Scherr model

$$C\left(\left|\mathbf{r}_1-\mathbf{r}_2\right|\right)=\exp\left(-\left|\mathbf{r}_1-\mathbf{r}_2\right|^2/\sigma^2\right) \quad (3)$$

where σ is the transverse coherence length.

For the quasi-monochromatic optical field, the CSD of a partially coherent beam with noncanonical vortex pairs at $z = 0$ can be written as

$$W(\mathbf{r}_1,\mathbf{r}_2,0)=\left\langle E^*\left(\mathbf{r}_1,0\right)E\left(\mathbf{r}_2,0\right)\right\rangle \quad (4)$$

where < > denotes the ensemble averaging, $k = 2\pi/\lambda$ is the wavenumber with λ being the beam wavelength.

In the presence of free space, the CSD of a partially coherent beam containing noncanonical vortex pairs along the z-axis follows a generalized Huygens-Fresnel integral with the expression

$$\begin{aligned}&W(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)\\&=\left(\frac{k}{2\pi B}\right)^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}W(\mathbf{r}_1,\mathbf{r}_2,0)\\&\times\exp\left[-\frac{ik}{2B}\left[A\mathbf{r}_1^2-2\left(\rho_{1x}r_{1x}+\rho_{1y}r_{1y}\right)+D\boldsymbol{\rho}_1^2\right]\right]\\&\times\exp\left[\frac{ik}{2B}\left(A\mathbf{r}_1^2-2\left(\rho_{2x}r_{2x}+\rho_{2y}r_{2y}\right)+D\boldsymbol{\rho}_1^2\right)\right]d\mathbf{r}_1d\mathbf{r}_2\end{aligned} \quad (5)$$

where $\boldsymbol{\rho}_1 = (\rho_{x1}, \rho_{y1})$ and $\boldsymbol{\rho}_2 = (\rho_{x2}, \rho_{y2})$ are two arbitrary transverse position vectors in the receiver plane ($z > 0$).

Recall the following equations

$$\left(x+iy\right)^l=\sum_{s=0}^{l}\frac{l!}{s!\left(l-s\right)!}x^{l-s}\left(iy\right)^s \quad (6)$$

$$\begin{aligned}&\int_{-\infty}^{+\infty}x^n\exp\left(-px^2+2qx\right)dx=n!\exp\left(\frac{q^2}{p}\right)\\&\times\left(\frac{q}{p}\right)^n\sqrt{\frac{\pi}{p}}\sum_{k=0}^{\left[\frac{n}{2}\right]}\frac{1}{k!\left(n-2k\right)!}\left(\frac{p}{4q^2}\right)^k\end{aligned} \quad (7)$$

According to Eqs. (6)–(7), the CSD on the receiving plane can be derived from Eq. (5).

We select chiral media for propagation. Optical rotation or optical activity can be found when a linear-polarized wave travels through a chiral medium, where the linear-polarized wave at the boundary can be separated into the left circularly polarized (LCP) wave and the right circularly polarized (RCP) wave. In general, the refractive indices of the LCP and RCP waves in a chiral medium are $n_{(l)} = n_0/(1 + n_0k\xi)$ or $n_{(l)} = n_0/(1 - n_0k\xi)$, respectively, where n_0 denotes the original refractive index and ξ represents the chirality parameter.

The ABCD optical transfer matrix in a chiral medium is expressed by

$$\begin{pmatrix}A & B\\ C & D\end{pmatrix}=\begin{pmatrix}1 & z/n'\\ 0 & 1\end{pmatrix} \quad (8)$$

where n' can be replaced by n(l), n(r) for the LCP or RCP wave, respectively. On substituting Eq. (8) into Eq. (5), one can obtain the electric field of LCP and RCP waves propagating along a chiral medium, respectively.

3. Numerical simulation and analysis

In this section, numerical simulation is used to study beam propagation in the chiral media, and the initial parameters are set as $\lambda = 632.8$nm, $w_0 = 1$mm, $\sigma = w_0$, $d = 0.5w_0$, $l_1=l_2=+2$, $n_0 = 3$, and Rayleigh length $z_0 = kw_0^2/2$ unless otherwise stated.

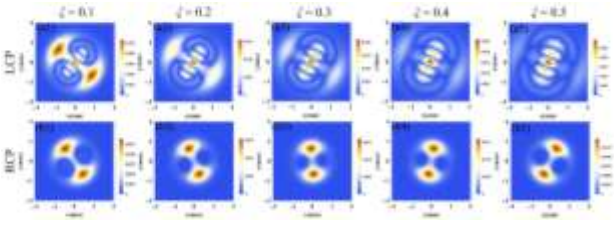

Figure 1. CCF distribution of a partially coherent beam with a canonical vortex-pair for different chirality parameters at $z = 1\ z_0$. (a1)-(a5): LCP; (b1)-(b4): RCP

The cross-correlation (CCF) at the receiver plane can be derived by $\chi(\boldsymbol{\rho}) = W(\boldsymbol{\rho}, -\boldsymbol{\rho}, z) = 0$. Figure 1 illustrates the evolution of CCF under varying chiral parameters and chiral media. The figure reveals that when the chiral medium is LCP, the number of rings in CCF consistently equals the sum of topological charges, $N = l_1 + l_2$. As the chiral medium increases, the ring structure of CCF gradually separates, with the originally nested double rings splitting into four independent rings. However, when the chiral medium is RCP, the ring number $N = 4$ is maintained at $\xi = 0.1$. As the chiral medium gradually increases, the ring structure gradually disappears between $\xi = 0.2$ and $\xi = 0.4$. At $\xi = 0.5$, the ring number recovers to $N = 4$.

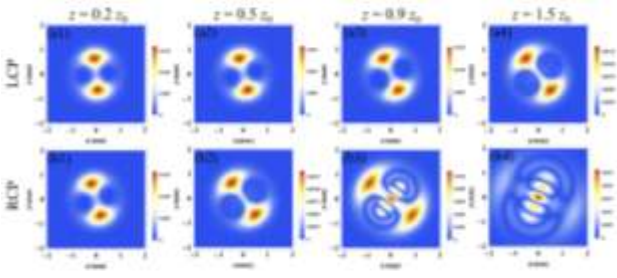

Figure 2. CCF distribution of a partially coherent beam with a canonical vortex-pair for different propagation distances and chiral media at $\xi = 0.16$. (a1)-(a4): LCP; (b1)-(b4): RCP.

Figure 2 illustrates the evolution of the CCF across varying propagation distances and chiral media. The figure reveals that when the chiral medium is LCP, the ring-like structure of the CCF does not manifest at shorter propagation distances. It gradually emerges after propagating a certain distance, appearing as two separate nested elliptical structures. In contrast, when the chiral medium is RCP, the ring-like structure of CCF becomes apparent at even shorter propagation distances. As the propagation distance increases, the two separate nested elliptical structures split into four independent ring-like structures.

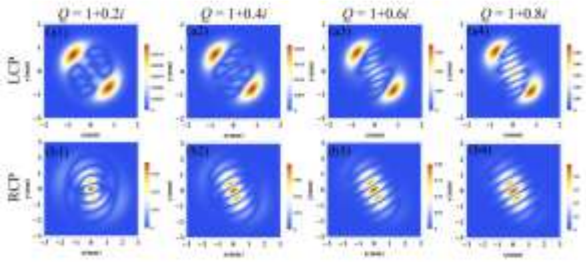

Figure 3. CCF distribution of a partially coherent beam with a noncanonical vortex-pair for different noncanonical strengths at ξ = 0.16. (a1)-(a4): LCP; (b1)-(b4): RCP.

Figure 3 illustrates the evolution of CCF under varying noncanonical strengths and chiral media. The figure reveals that when the chiral medium is LCP, the ring structure of CCF remains separated throughout. As the noncanonical strengths increase, this structure gradually transforms from irregular shapes into elliptical configurations. When the chiral medium is RCP, at low noncanonical strength, the CCF ring structure exhibits nested elliptical configurations. As noncanonical strength increases, these ring structures progressively separate, achieving complete separation at higher noncanonical strengths.

4. CONCLUSION

We have systematically investigated the propagation characteristics and the evolution of the CCF of partially coherent beams embedded with noncanonical vortex pairs through chiral media. Through theoretical modeling and numerical simulations, several key insights have been revealed. The handedness of the chiral medium—LCP or RCP—plays a decisive role in determining the structure and evolution of the CCF. In LCP media, the number of rings in the CCF consistently equals the sum of the topological charges. In contrast, RCP media induce more complex structural transformations, including ring separation and reconstruction. Moreover, both the propagation distance and the chirality parameter significantly influence the morphological development of the CCF. The noncanonical strength also serves as an effective degree of freedom for modulating the coherence structure, enabling transitions from nested elliptical configurations to well-separated ring patterns, particularly in RCP media. These findings not only deepen our understanding of the singular optics of structured partially coherent beams in complex media but also provide valuable guidance for designing optical systems for applications in communication, sensing, and manipulation utilizing customized coherence and phase properties.

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