A Time-Frequency Analysis Method Based on Synchroextracting Transform

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Abstract: The Synchroextracting Transform (SET) is a novel and powerful time-frequency analysis technique designed to overcome the resolution limitation imposed by the Heisenberg uncertainty principle in traditional methods. Unlike conventional approaches that retain the entire time-frequency spectrum, SET employs a synchroextracting operator to identify and preserve only the most energy-concentrated time-frequency coefficients associated with the signal's instantaneous frequency. This process effectively filters out smeared and diffused energy, resulting in a highly sparse and sharpened time-frequency representation. This paper elaborates on the fundamental principle of SET and validates its performance using synthetic non-stationary signals. The results demonstrate that SET achieves superior energy concentration and provides exceptional resolution in distinguishing signal components that are otherwise blurred in Short-Time Fourier Transform (STFT) and S-Transform analyses. The method shows significant potential for high-precision signal characterization in various engineering and geophysical applications.

Keywords: time-frequency analysis; Synchroextracting Transform; instantaneous frequency; resolution enhancement; signal processing

1. INTRODUCTION

TIME-FREQUENCY analysis (TFA) serves as a cornerstone for processing and interpreting non-stationary signals whose frequency content changes over time [1]. Classical methods like the Short-Time Fourier Transform (STFT) are inherently constrained by the Heisenberg uncertainty principle, which mandates a trade-off between temporal and spectral resolution. While subsequent advancements, such as the S-Transform [2] and wavelet transform [3], offer adaptive windows and multi-resolution analysis, they often still produce blurred or "smeared" energy distributions in the time-frequency plane for complex multi-component signals. This blurring effect obscures critical signal features, such as closely spaced instantaneous frequency ridges, limiting the accuracy of signal decomposition and feature extraction [4]. To address this fundamental challenge, the Synchroextracting Transform (SET) was introduced as a novel post-processing technique [5]. Its core philosophy is to move beyond representing all energy and instead extract only the most physically meaningful components, resulting in a highly sparse and sharpened time-frequency representation. This paper details the theory behind SET and presents a comparative analysis to underscore its superior performance in achieving a high-resolution time-frequency representation for seismic signals.

2. THEORY

THE SET is a post-processing method applied to the initial STFT result. It aims to sharpen the time-frequency representation by retaining only the coefficients that are most pertinent to the signal's intrinsic time-varying frequency characteristics.

The process begins with the STFT, which provides a preliminary time-frequency distribution. For a continuous signal x(t), its STFT is defined as:

$$F(t,\omega) = \int_{-\infty}^{\infty} x(\tau)g(\tau-t)e^{-j\omega\tau}d\tau \quad (1)$$

where g(t) is a short-time analysis window (e.g., a Gaussian window) centered at time t.

The key step in SET is the accurate estimation of the instantaneous frequency (IF) from the STFT phase spectrum. The IF $\omega_0(t,\omega)$ at a specific point (t,ω) in the time-frequency plane is calculated using the following formula:

$$\omega_0(t,\omega) = -j \frac{\frac{\partial F(t,\omega)}{\partial t}}{F(t,\omega)} \quad (2)$$

This derivative-based estimate pinpoints the true frequency trajectory of the signal components.

The synchroextracting operator (SEO) is a binary mask designed to filter the STFT result. It is defined based on the proximity of the frequency coordinate ω to the estimated instantaneous frequency $\omega_0(t,\omega)$:

$$\Gamma(t,\omega) = \delta(\omega - \omega_0(t,\omega)) \quad (3)$$

In discrete implementation, this is approximated with a threshold ϵ:

$$\Gamma(t,\omega) = \{1, if \mid \omega - \omega_0(t,\omega) \mid \leq \grave{\alpha}; 0, otherwise\} \quad (4)$$

The high-resolution SET spectrum $S(t,\omega)S(t,\omega)$ is finally obtained by applying the SEO to the modulus of the STFT:

$$S(t,\omega) = \mid F(t,\omega) \mid \cdot \Gamma(t,\omega) \quad (5)$$

This operation effectively preserves the time-frequency coefficients that lie precisely on the instantaneous frequency ridges of the signal while discarding the diffuse energy, resulting in a sharply concentrated and sparse time-frequency representation.

3. NUMERICAL EXAMPLES

To comprehensively evaluate the high-resolution time-frequency analysis capability of the Synchroextracting Transform (SET), a synthetic seismic signal was meticulously constructed. This signal was generated by convolving a 30 Hz Ricker wavelet, a standard model for seismic wavelets, with a predefined reflectivity series containing several distinct events, including two closely spaced reflections. The resultant synthetic seismogram was then subjected to time-frequency analysis using three different methods: the conventional Short-Time Fourier Transform (STFT), the S-transform, and the proposed SET. The corresponding time-frequency representations (TFRs) are presented in Fig. 1(b), (c), and (d), respectively. A comparative analysis of these results reveals distinct performance characteristics. The STFT spectrum (Fig. 1(b)) exhibits broad and smeared energy distributions, a consequence of the fixed window length and the inherent trade-off of the Heisenberg uncertainty principle. While the S-transform (Fig. 1(c)), with its frequency-dependent window, offers improved resolution over the STFT, it still fails to fully concentrate the energy and cleanly separate the two closely spaced events. In stark contrast, the SET (Fig. 1(d)) demonstrates a remarkable ability to sharpen the TFR. It successfully resolves the individual seismic events with unprecedented clarity, effectively concentrating the spectral energy around the dominant 30 Hz frequency and producing thin, well-defined time-frequency ridges. This outcome unambiguously validates SET's superior performance in achieving high-concentration time-frequency analysis, which is crucial for resolving fine-scale geological features.
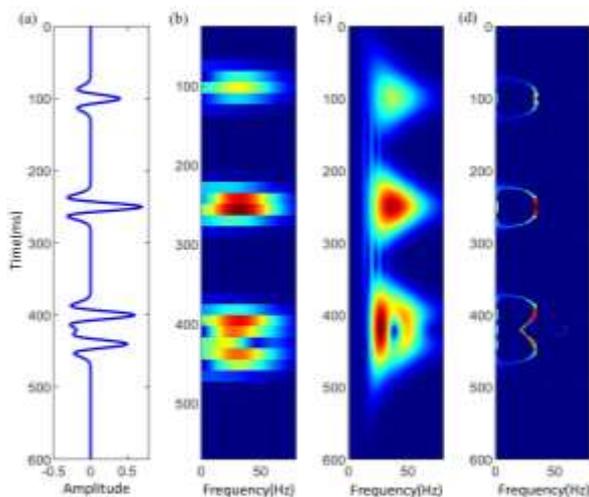

Figure 1. Time-frequency analysis using different methods.
(a) Original signal. (b) STFT. (c) S-Transform. (d) SET.

4. CONCLUSION

This paper has elaborated on the principle and demonstrated the performance of the Synchroextracting Transform, a high-resolution time-frequency analysis method. By leveraging a synchroextracting operator derived from instantaneous frequency estimation, SET successfully overcomes the smearing effect that plagues conventional methods. The numerical example confirms that SET produces an exceptionally sparse and concentrated time-frequency distribution, enabling the clear separation and precise tracking of individual signal components. This superior resolution makes SET a highly valuable tool for applications requiring detailed time-frequency feature analysis, such as fault diagnosis, radar signal analysis, and seismic interpretation. Future work will explore its robustness in highly noisy environments and its integration with machine learning for automatic feature recognition.

5. REFERENCES

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