

Seismic Random Noise Attenuation via Frequency-Binned Successive Variational Mode Decomposition and Multichannel Singular Spectrum Analysis

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Abstract: To address the limitations of variational mode decomposition (VMD) in seismic denoising—namely, the necessity of presetting the number of modes and the disruption of event lateral continuity caused by trace-by-trace processing, a denoising method based on frequency-binned successive variational mode decomposition and multichannel singular spectrum analysis (SVMD-MSSA) is proposed. First, 1-D SVMD is utilized to adaptively extract intrinsic mode functions trace by trace, eliminating the need to preset the mode number based on prior knowledge. Furthermore, a frequency-binning spatial alignment strategy is formulated to route the decomposed modes of each trace into unified global wavefield sections according to their physical center frequencies, thereby resolving mode misalignment and lateral discontinuity. Finally, adaptive rank-reduction filtering is implemented via frequency-domain 2-D MSSA. In tests on a complex model with crossing and curved events under 0 dB noise, the reconstructed signal-to-noise ratio (SNR) of the proposed method outperforms both conventional MSSA and VMD-MSSA, with no leakage of valid wavefields in the difference residual profile.

Keywords: Seismic random noise; Successive variational mode decomposition (SVMD); Multichannel singular spectrum analysis (MSSA); Frequency-binning alignment; Rank reduction

1. INTRODUCTION

Raw seismic exploration data are invariably contaminated by random noise, which severely degrades the lateral continuity of seismic events and undermines subsequent imaging and reservoir interpretation [1]. Integrating time-frequency mode decomposition with multichannel low-rank filtering has emerged as a prominent technical route for random noise suppression [2]. Among these approaches, variational mode decomposition coupled with multichannel singular spectrum analysis (VMD-MSSA) decomposes broadband seismic data into band-limited intrinsic mode functions (IMFs), facilitating targeted filtering within individual sub-bands. Nevertheless, standard VMD necessitates a predefined number of modes K , where improper presets frequently cause mode mixing, under-decomposition, or spurious noisy modes [3]. Furthermore, VMD concurrently solves a multi-mode optimization problem, incurring considerable computational burdens and lateral discontinuity due to trace-by-trace initialization sensitivity [4]. Although multivariate VMD (MVMD) enforces spatial consistency, its multidimensional matrix inversions incur prohibitive computational costs [5]. To address these challenges, this study incorporates successive variational mode decomposition (SVMD) [6]. SVMD extracts modes sequentially through single-mode sub-problems, automatically terminating without presetting K while demonstrating rapid convergence and robustness against initial center frequency variations [7]. To mitigate the loss of spatial continuity in trace-by-trace workflows, a frequency-binning alignment strategy is introduced [8], establishing a unified SVMD-MSSA processing framework that balances computational efficiency and structural preservation.

2. THEORY

2.1 SVMD

For an individual seismic trace $f(t)$, SVMD formulates the signal as the sum of the L -th mode to be extracted $u_L(t)$ and the residual signal $f_r(t)$:

$$f(t) = u_L(t) + f_r(t) = u_L(t) + \sum_{i=1}^{L-1} u_i(t) + f_u(t) \quad (1)$$

where $f_u(t)$ denotes the unprocessed signal component. The constrained optimization problem for extracting $u_L(t)$ is established as:

$$\min_{u_L, \omega_L, f_r} \{ \alpha J_1 + J_2 + J_3 \} \quad \text{s.t.} \quad u_L(t) + f_r(t) = f(t) \quad (2)$$

Here, J_1 enforces spectral compactness of $u_L(t)$ around its center frequency ω_L ; J_2 and J_3 serve as penalty criteria minimizing spectral overlap with the residual signal and previously extracted modes, respectively; and α is a balancing parameter. The alternating direction method of multipliers (ADMM) iteratively updates the mode spectrum $\hat{u}_L(\omega)$ and center frequency ω_L until the residual energy satisfies the predefined termination threshold.

2.2 Frequency-Binning Spatial Alignment

Assembling IMFs directly across traces according to their extraction order inevitably introduces mode misalignment due to noise perturbations, forming artificial discontinuities. To resolve this, a set of global frequency bins $\mathcal{B}_m = [\Omega_m^{\min}, \Omega_m^{\max})$ is defined according to the wavelet's spectral footprint. The k -th mode $u_{i,k}(t)$ of trace i is routed and accumulated into the corresponding wavefield profile $\mathbf{S}_m(t, i)$ based on its estimated center frequency $\omega_{i,k}$:

$$\mathbf{S}_m(t, i) = \sum_{k, \omega_{i,k} \in \mathcal{B}_m} u_{i,k}(t) \quad (3)$$

This operation eliminates inter-trace mode jumping and restores the lateral coherence of each sub-band profile $\mathbf{S}_m$.

2.3 MSSA

Fast Fourier transform (FFT) along the temporal axis is performed on each aligned slice $\mathbf{S}_m$ to obtain $\hat{\mathbf{S}}_m(\omega, x)$. At a specific frequency slice ω_0 , spatial samples $\mathbf{d} = [\hat{\mathbf{S}}_m(\omega_0, 1), \dots, \hat{\mathbf{S}}_m(\omega_0, N_x)]$ are arranged into a block Hankel matrix $\mathbf{H} \in \mathbb{C}^{L_h \times (N_x - L_h + 1)}$:

$$\mathbf{H} = \begin{bmatrix} d_1 & d_2 & \cdots & d_{N_x - L_h + 1} \\ d_2 & d_3 & \cdots & d_{N_x - L_h + 2} \\ \vdots & \vdots & \ddots & \vdots \\ d_{L_h} & d_{L_h + 1} & \cdots & d_{N_x} \end{bmatrix} \quad (4)$$

Spatially coherent events render $\mathbf{H}$ low-rank. Truncated singular value decomposition (TSVD) retains the dominant a_m singular values, discarding trailing components dominated by incoherent noise, followed by anti-diagonal averaging to reconstruct the spatial vector:

$$\mathbf{H}_{\text{filtered}} = \sum_{r=1}^{a_m} \sigma_r \mathbf{u}_r \mathbf{v}_r^H \quad (5)$$

Filtered time-domain sections are subsequently synthesized by inverse FFT and summed across all frequency bands to yield the denoised seismic volume.

3. NUMERICAL EXAMPLES

To evaluate the denoising and amplitude-preservation capability of the proposed method, a synthetic 2-D record with eight complex reflection events (horizontal, dipping, and curved parabolic events; 60 traces, 0.5 s duration, 1 ms dt, 40 Hz Ricker wavelet) was generated (Fig. 1(a)). Gaussian white noise was added to produce a 0.00 dB noisy record (Fig. 1(b)). Denoising performance is measured by signal-to-noise ratio:

$$\text{SNR} = 10 \log_{10} \left(\frac{\|\mathbf{D}_{\text{clean}}\|_F^2}{\|\mathbf{D}_{\text{denoised}} - \mathbf{D}_{\text{clean}}\|_F^2} \right) \quad (6)$$

Denoised profiles and difference residuals are shown in Fig. 1, with reconstructed SNRs indicated in subfigure titles.

Single-scale MSSA yields an output SNR of 7.12 dB (Fig. 1(c)), while sub-band VMD-MSSA achieves 8.39 dB (Fig. 1(e)). The proposed SVMD-MSSA operates autonomously without presetting mode numbers and reaches the highest SNR of 8.84 dB (Fig. 1(g)), offering improvements of 1.72 dB and 0.45 dB over MSSA and VMD-MSSA, respectively.

Structurally, MSSA over-smooths dipping events, and VMD-MSSA exhibits slight trace-to-trace jitter due to independent frequency updates. Conversely, SVMD-MSSA accurately reconstructs curved parabolic apexes and dipping intersections with smooth lateral continuity. In the residual profiles, MSSA (Fig. 1(d)) and VMD-MSSA (Fig. 1(f)) show noticeable structural footprints around reflection apexes and crossings. In contrast, the SVMD-MSSA residual (Fig. 1(h)) displays purely random Gaussian noise without coherent signal leakage, confirming superior amplitude preservation.

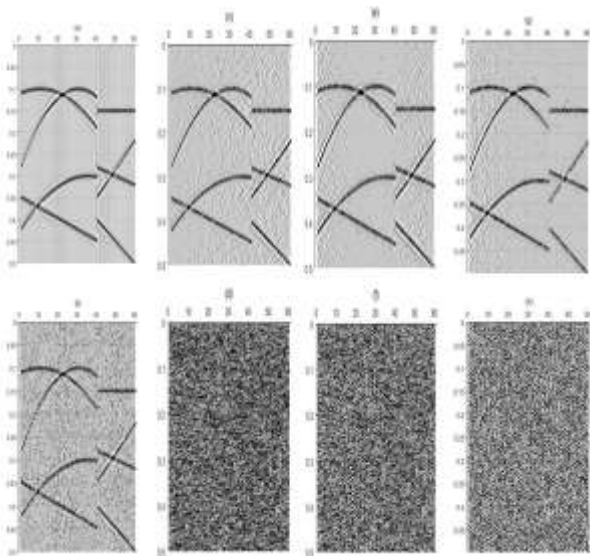


Figure 1. Comparison of denoising results and difference residuals on the synthetic seismic data: (a) Noise-free data; (b) Noisy record (SNR = 0.00 dB); (c) Filtered by MSSA (SNR = 7.12 dB); (d) Difference residual of MSSA; (e) Filtered by VMD-MSSA (SNR = 8.39 dB); (f) Difference residual of VMD-MSSA; (g) Filtered by the proposed SVMD-MSSA (SNR = 8.84 dB); (h) Difference residual of the proposed SVMD-MSSA.

4. CONCLUSION

This paper presents a hybrid seismic random noise attenuation scheme based on successive variational mode decomposition and multichannel singular spectrum analysis (SVMD-MSSA). The primary conclusions are as follows:

1. Replacing joint optimization in VMD with the sequential extraction mechanism of SVMD eliminates the requirement of presetting mode number K , avoiding under- or over-decomposition.
2. The frequency-binning spatial alignment strategy circumvents mode misalignment caused by trace-by-trace processing, ensuring that decomposed sub-bands comply with the lateral low-rank assumptions of subsequent MSSA filtering.
3. Numerical tests verify that SVMD-MSSA achieves an output SNR of 8.39 dB without structural leakage in the residual profile, highlighting its practical value for amplitude-preserved seismic data processing.

5. REFERENCES

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