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Fuzzy traditional EPQ model allows backorders with planned shortages by yager's ranking method

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Abstract: The nature of this paper reports of an investigation of a set of continuous time ,constant demand inventory models under the condition of yield uncertainty. We consider a classical EPQ model, that models are lot sizing models whose objective is the determination of fuzzy optimal production order and shipment quantity.In this paper, we discuss the inventory problem with fuzzy backorder. Yager's ranking method for fuzzy numbers is utilized to find the inventory policy in terms of the fuzzy total cost. Finally a numerical example is given to illustrate the model.

Keywords: Fuzzy number, inventory, backorder, yager's method

1. INTRODUCTION

Within the context of traditional inventory models, the pattern of demands is either deterministic or uncertain. In practice, the latter corresponds more to the real-world environment. To solve these inventory problems with uncertain demands, the classical inventory models usually describe the demands as certain probability distributions and then solve them. However, some times, demands may be fuzzy, and more suitably described by linguistic term rather than probability distributions. If the traditional inventory theories can be extended to fuzzy senses, the traditional inventory models would have wider applications. Usually, inventory systems are characterized by several parameters such as cost coefficients, demands etc. Accordingly, most of the inventory problems under fuzzy environment can be addressed by fuzzying these parameters. For instance, Park [8] discussed the EOQ model with fuzzy cost coefficients. Ishii and Konno [3], Petrovic et. al. [9], and Kao and Hsu [4] investigated the Newsboy inventory model with fuzzy cost coefficients and demands respectively. Roy and Maiti [10] developed a fuzzy EOQ model with a constraint of fuzzy storage capacity. Chang [1] construct a fuzzy EOQ model with fuzzy defective rate and fuzzy demand. Yao and Chiang [13] compare the EOQ model with fuzzy demand and fuzzy holding cost in different solution methods. Kao and Hsu [5] find the lot size-reorder point model with fuzzy demand. Besides, there is another kind of studies which fuzzes the decision variables of inventory models. For example: Yao and Lee [15] developed the EOQ model with fuzzy ordering quantities; Chang and Yao [2] investigated the EOQ model with fuzzy order point; Wen-Kai K. Hsu and Jun-Wen Chen [11] studied Fuzzy EOQ model with stock out. Madhu & Deepa [7] developed an EOQ model for deteriorating items having exponential declining rate of demand under inflation & shortage. Kun-Jen Chung, Leopoldo Eduardo Cárdenas-Barrón [6] compare the complete solution procedure for the EOQ and EPQ inventory models with linear and fixed backorder costs. Recently W. Ritha etal. [14] fuzzified EOQ Model with one time discount offer allowing back.

In this paper, we discuss the inventory problem with fuzzy back order. The decision variables are the ordering quantity Q and the back order quantity S. The approach of this paper is to find the optimal order quantity Q* with the minimum cost determined from Yager's ranking method.

2.PRELIMINARIES:

Definition :

A fuzzy set $\hat{A}$ is defined by

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X, \mu_{\tilde{A}} \in [0,1]\}.$$

In the pair $(x, \mu_{\tilde{A}}(x))$, the first element x belong to the classical set A, the second element $\mu_{\tilde{A}}(x)$ belong to the interval $[0,1]$, called membership function or grade membership. The membership function is also a degree of compatibility or a degree of truth of x in $\tilde{A}$.

Definition : α**-cut**

An α-cut of a fuzzy set $\hat{A}$ is a crisp set A_α,that contains all the elements of universal set X having a membership grade in A greater than (or) equal to the specific value of α.

i.e., $A_\alpha = \{x \in X \mid \mu_{\tilde{A}}(x) \geq \alpha\}$

Generalised Fuzzy number

Any fuzzy subset of the real line R, whose membership function satisfies the following conditions is a generalized fuzzy number.

(i) $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval $[0,1]$.
(ii) $\mu_{\tilde{A}}(x)$=0, $-\infty \leq x \leq a$,
(iii) $\mu_{\tilde{A}}(x) = L_{(x)}$ is strictly increasing on $[a_1, a_2]$
(iv) $\mu_{\tilde{A}}(x) = 1, a_1 \leq x \leq a_2$.
(v) $\mu_{\tilde{A}}(x)$ = R(x) is strictly decreasing on $[a_1, a_2]$
(vi) $\mu_{\tilde{A}}(x) = 0, , a_4 \leq x \leq \infty$.

Where a_1, a_2, a_3 and a_4 are real numbers.

3. YAGER'S RANKING METHOD:

If the α-cut of any fuzzy number $\tilde{A}$ is $\frac{1}{2}\int_0^1 [A_L(\alpha) + A_U(\alpha)]\ d\alpha$.

4. MODEL DEVELOPMENT

Notations Used:

D - Annual Demand in units

K - Ordering cost per order

c_h - Holing cost per unit

c_b - backordering cost per unit per year

Q - lot size per order

S - size of the back order quantity

Q* - optimal value of Q

Mathematical Model

Consider the traditional Economic Order Quantity (EOQ) model that allows backordering with the following total annual cost function

$$C_{Trad}(Q,S) = \frac{D}{Q}k + \frac{(Q-S)^2}{2Q}C_h + \frac{S^2}{2Q}C_b \qquad (1)$$

The objective is to find the optimal order quantity which minimize the total cost

The necessary conditions for minimum

$$\frac{\partial C_{Trad}(Q,S)}{\partial Q} = -\frac{DK}{Q^2} + \frac{C_h}{2} - \frac{S^2C_h}{2Q^2} + \frac{S^2C_b}{2Q^2} = 0$$

Differentiate (1) partially w.r.to S, we obtain

$$\frac{\partial C_{Trad}(Q,S)}{\partial S} = \frac{SC_h}{Q} + C_h + \frac{SC_b}{Q} = 0$$

$$S = \frac{C_hQ}{C_h + C_b} \qquad (2)$$

Substitute S in (1), Hence the optimal order quantity is

$$Q^* = \sqrt{\frac{2DK(C_h + C_b)}{C_hC_b}}$$

The EOQ model with back order and fuzzy demands

Let $\tilde{D}, \tilde{K}, \tilde{C_b}, \tilde{C_h}$ be the trapezoidal numbers and they are defined as follows i.e, they are described by the α-cuts.

$$K(\alpha_K) = [L_K^{-1}(\alpha_K), R_K^{-1}(\alpha_K)]$$

$$D(\alpha_D) = [L_D^{-1}(\alpha_D), R_D^{-1}(\alpha_D)]$$

$$C_h(\alpha_{C_h}) = [L_{C_h}^{-1}(\alpha_{C_h}), R_{C_h}^{-1}(\alpha_{C_h})]$$

$$C_b(\alpha_{C_b}) = [L_{C_b}^{-1}(\alpha_{C_b}), R_{C_b}^{-1}(\alpha_{C_b})]$$

Now

$C_{Trad}(Q,S) = \frac{D}{Q}k + \frac{(Q-S)^2}{2Q}C_h + \frac{S^2}{2Q}C_b$ can be rewritten as

$$\tilde{C}_{Trad}(Q,S) = \frac{\tilde{D}}{Q}\tilde{k} + \frac{(Q-S)^2}{2Q}\tilde{C_h} + \frac{S^2}{2Q}C_b$$

Yager's ranking index can be derived as

$$\tilde{C}_{Trad}(Q,S) = \frac{K_1(\alpha_D, \alpha_K)}{Q} + \frac{(Q-S)^2}{2Q}K_2(\alpha_{C_h}) + \frac{S^2}{2Q}K_3(\alpha_{C_b})$$

Where,

$$K_1(\alpha_D, \alpha_K) = \frac{1}{4}\left\{\int_0^1 L_D^{-1}(\alpha_D)d\alpha_D \cdot \int_0^1 L_K^{-1}(\alpha_K)d\alpha_K + \int_0^1 R_D^{-1}(\alpha_D)d\alpha_D \cdot \int_0^1 R_K^{-1}(\alpha_K)d\alpha_K\right\}$$

$$K_2(\alpha_{C_h}) = \frac{1}{2}\left\{\int_0^1 L_{C_h}^{-1}(\alpha_{C_h})d\alpha_{C_h} + \int_0^1 R_{C_h}^{-1}(\alpha_{C_h})d\alpha_{C_h}\right\}$$

$$K_3(\alpha_{C_b}) = \frac{1}{2}\left\{\int_0^1 L_{C_b}^{-1}(\alpha_{C_b})d\alpha_{C_b} + \int_0^1 R_{C_b}^{-1}(\alpha_{C_b})d\alpha_{C_b}\right\}$$

Hence the optimal order quantity is

$$Q^* = \frac{2K_1(K_2 + K_3)}{K_2K_3}$$

NUMERICAL VALIDATION:

To validate the proposed model, consider the data, $D = 1200, K = 100, C_h = 25, C_b = 50$ and hence by graded mean method

$\tilde{D}$ = (1000,1100,1300,1400)

$\tilde{K}$ = (98,99,101,102)

$\tilde{C_h}$ = (15,20,30,35)

$\tilde{C_b}$ = (40,45,55,60)

Thus

$D(\alpha_D)$= (1000+ 100α, $1400 - 100\alpha$)

$K(\alpha_K) = (98 + 5\alpha, 102 - 5\alpha)$

$C_h(\alpha_{C_h}) = (15 + 5\alpha, 35 - 5\alpha)$

$C_b(\alpha_{C_b}) = (40 + 5\alpha, 60 - 5\alpha)$

$$K_1(\alpha_D, \alpha_K) = \frac{1}{4}\left\{\int_0^1 (1000 + 100\alpha)d\alpha . \int_0^1 (98 + 5\alpha)d\alpha + \int_0^1 (1400 - 100\alpha)d\alpha . \int_0^1 (102 - 5\alpha)d\alpha\right\}$$

$K_1(\alpha_D, \alpha_K)$ = 60112.5

$K_2(\alpha_{C_h}) = \frac{1}{2}\left\{\int_0^1 (15 + 5\alpha)d\alpha + \int_0^1 (35 - 5\alpha)\, d\alpha\right\}$

$K_2(\alpha_h)$ = 25

$K_3(\alpha_{C_b}) = \frac{1}{2}\left\{\int_0^1 (40 + 5\alpha)d\alpha + \int_0^1 (60 - 5\alpha)\, d\alpha\right\}$

$K_3(\alpha_{C_b}) = 50$

Optimal Order quantity $Q = \sqrt{\frac{2DK(C_h + C_b)}{C_h C_b}}$=120 units

Optimal total cost $C_{Trad}(Q, S) = 2000$

Fuzzy Optimal Order quantity $Q^* = \sqrt{\frac{2K_1(K_2+K_3)}{K_2K_3}}$ = 85 units

Fuzzy Optimal total cost $C_{Trad}(Q^*, S^*) = 1475.588$

CONCLUSION:

The purpose of this paper is to study the inventory models under fuzzy environment. This fuzzy model assists in determining the optimal expected total cost per cycle amidst the existing fluctuations. In this paper the Yager's ranking method of optimization is employed.

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