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Successive approximation of neutral stochastic functional differential equations with infinite delay and Poisson jumps

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Abstract: We establish results concerning the existence and uniqueness of solutions to neutral stochastic functional differential equations with infinite delay and Poisson jumps in the phase space $C((-\infty,0];R^d)$ under non-Lipschitz condition with Lipschitz condition being considered as a special case and a weakened linear growth condition on the coefficients by means of the successive approximation. Compared with the previous results, the results obtained in this paper is based on a other proof and our results can complement the earlier publications in the existing literatures.

Keywords: neutral stochastic functional differential equations; Poisson jumps; infinite delay; successive approximation; Bihari's inequality

1. INTRODUCTION

Stochastic differential equations are well known to model problems from many areas of science and engineering, wherein, quite often the future state of such systems depends not only on the present state but also on its past history (delay) leading to stochastic functional differential equations (SFDEs) and it has played an important role in many ways such as the model of the systems in physics, chemistry, biology, economics and finance from various points of the view (see, e.g. [1,2] and the references therein).

Recently, SFDEs with infinite delay on the space $BC((-\infty,0];R^d)$, which denotes the family of bounded continuous R^d -value functions φ defined on $(-\infty,0]$ with norm $\|\varphi\| = sup_{\theta\in(-\infty,0]}|\varphi(\theta)|$ have been extensively studied by many authors, for instance, in [3], Wei and Wang studied the existence and uniqueness of the solution for SFDEs with infinite delay under uniform Lipschitz condition and a weakened linear growth condition, Zhou et al. [4] investigated the stability of the solutions for SFDEs with infinite delay, and in 2010, Xu and Hu [5] have proved the existence and uniqueness of the solution for neutral SFDEs with infinite delay in abstract space. Note that, the results on the existence and uniqueness of the solution for the above equations is obtained if the coefficient of the equation is assumed to satisfy the Lipschitz condition and the linear growth condition. Furthermore, on the neutral SFDEs with delay, once can see monograph [1] and the references therein for details.

On the other hand, the study of neutral SFDEs with Poisson jumps processes also have begun to gain attention and strong growth in recent years. To be more precise, in 2009, Luo and Taniguchi [6] considered the existence and uniqueness of mild solutions to stochastic evolution equations with finite delay and Poisson jumps by the Banach fixed point theorem, in 2010, Boufoussi and Hajji [7] proved the existence and uniqueness result for a class of neutral SFDEs driven both by the cylindrical Brownian motion and by the Poisson processes in a Hilbert space with non-Lipschitzian coefficients by using successive approximation, in 2012, Cui and Yan [8] studied the existence and uniqueness of mild solutions to stochastic evolution equations with infinite delay and Poisson jumps in the phase space $BC((-\infty,0];H)$, and also in 2012, Tan et al. [9] established the existence and uniqueness of solutions to neutral SFDEs with Poisson Jumps. However, until now, there is no work on the existence and uniqueness of the solution to neutral SFDEs with infinite delay and Poisson jumps in the phase space $C((-\infty,0];R^d)$ under non-Lipschitz condition and a weakened linear growth condition. Therefore, motivated by [5,8,9] in this paper, we will closes the gap and further perfects the theorem system of existence and uniqueness of the solution to the following d-dimensional neutral SFDEs with infinite delay and Poisson jumps:

$$d[x(t) - g(x_t)] = f(t, x_t)dt + \sigma(t, x_t)dB(t)$$

$$+h(t,x_t)dN(t), \qquad t\in[t_0,T], \qquad (1.1)$$

with an initial data

$$x_{t_0}=\varphi=\{\varphi(t):-\infty<t\le 0\} \qquad (1.2)$$

is an F_{t_0}-measurable $C((-\infty,0];R^d)$ -value random variable such that $\varphi\in M^2((-\infty,0];R^d)$, where $x_t=x(t+\theta),\ \theta\in(-\infty,0]$ can be considered as a $C((-\infty,0];R^d)$ - value stochastic process. Moreover, let the functions $g:\ C((-\infty,0];R^d)\to R^d$;

$$f,h:[t_0,T]\times\ C((-\infty,0];R^d)\to R^d;$$

$$\sigma:[t_0,T]\times\ C((-\infty,0];R^d)\to R^{d\times m}$$

all be Borel measurable.

The aim of our paper is to establish an existence and uniqueness results for solution of Eq.(1.1) with initial data (1.2) in the phase $C((-\infty,0];R^d)$ under non-Lipschitz condition and a weakened linear growth condition based on successive approximation method. Our main results rely essentially on techniques using a iterative scheme (see, [10]) which is partially different from the Picard iterative and Bihari's inequality. We will see that the proof of claim in the theorem of this paper is partially different and even simpler than the work has been previously published.

The rest of this paper is organized as follows: In Section 2, we will give some necessary notations, concepts and assumptions. Section 3 is devoted to prove the existence and uniqueness of Eq.(1.1) with initial data (1.2) under non-Lipschitz condition and a weakened linear growth condition. In the last section, concluding remarks are given.

2. PRELIMINARIES RESULTS

This section is concerned with some basic concepts, notations, definitions, lemmas and preliminary facts which are used through this article.

Let (Ω,F,P) be a complete probability space equipped with some filtration $\{F_t,t\ge0\}$ satisfying the usual conditions (i.e., it is right continuous and $\{F_t,t\ge0\}$ contains all P-null sets). Let $|\cdot|$ be the Euclidean norm in R^d i.e., $|x|=\sqrt{x^Tx},\quad x\in R^d$. If A is a vector or a matrix, its transpose is denoted by A^T. If A is a matrix, its trace norm is represented by $|A|=\sqrt{A^TA}$, while its operator norm is denoted $|A|=sup\{Ax:|x|=1\}$, (without any confusion with $\|\rho\|$). Without loss of generality, let t be a positive constant. Assume that B(t) is a m-dimensional Brownian motion defined on complete probability space, that is $B(t)=(B_1(t),B_2(t),\ldots,B_m(t))^T$, and N(t) is a scalar Poisson process with intensity λ. Assume that B(t) and N(t) are independent of $\{F_t,t\ge0\}$. Let $C((-\infty,0];R^d)$ denotes the family of all right-continuous functions with left-hand limits (cadlag) $(-\infty,0]$ to R^d. The space $C((-\infty,0];R^d)$ is assumed to be equipped with the norm $\|\rho\|=sup_{\theta\in(-\infty,0]}|\rho(\theta)|$. We denote by $\varphi\in M^2((-\infty,0];R^d)$ the family of all $\{F_t,t\ge0\}$ -measurable, R^d -valued process $\varphi(t)=\varphi(t,\omega)$, $t\in(-\infty,0]$ such that $E\int_{-\infty}^0|\varphi(t)|^2dt<\infty$. And let $L^p([a,b];R^d)$, $p\ge2$ be the family of R^d -valued F_t-adapted processess $\{\gamma(t):a\le t\le b\}$ such that $\int_a^b|\gamma(t)|^pdt<\infty$. Further, we consider the Banach space B_T of all R^d -valued F_t -adapted cadlag process x(t) defined on $(-\infty,T]$, T>0 such that

$$\|x\|_{B_T}^2:=E\left(\sup_{t\in(-\infty,T]}|x(t)|^2\right)<\infty.$$

For simplicity, we also have to denote by $a\wedge b:=\min\{a,b\}$ and $a\vee b:=\max\{a,b\}$.

Let us give the definition of solution for Eq.(1.1) with initial data (1.2).

Definition 2.1 An R^d -valued stochastic process x(t) defined on $-\infty<t\le T$ is called the solution of Eq.(1.1) with initial data (1.2), if it has the following properties:

(i) x(t) is continuous and $\{x(t):t_0\le t\le T\}$ is F_t -adapted;

(ii) $\{f(t,x_t)\}\in L^1([t_0,T];R^d)$, $\{\sigma(t,x_t)\}\in L^2([t_0,T];R^{d\times m})$, and $\{h(t,x_t)\}\in L^2([t_0,T];R^{d\times m})$;

(iii) $x_{t_0}=\varphi$ and for $t_0\le t\le T$,

$$x(t)=\varphi(0)-g(\varphi)+g(x_t)+\int_{t_0}^t f(s,x_s)ds$$
$$+\int_{t_0}^t\sigma(s,x_s)dB(s)+\int_{t_0}^t h(s,x_s)dN(s), \quad (2.1)$$

A solution x(t) is said to be unique if any other solution x*(t) is indistinguishable with x(t), that is

$$P\{x(t)=x^*(t),\ \text{for any } -\infty<t\le T\}=1.$$

In order to guarantee the existence and uniqueness of the solutions to Eq.(1.1) with initial data (1.2), we propose the following assumptions:

(H1) For all $\xi,\eta \in C((-\infty,0];R^d)$ and $t_0 \le t \le T$, it follows that

$$|f(t,\xi)-f(t,\eta)|^2 \vee |\sigma(t,\xi)-\sigma(t,\eta)|^2 \vee |h(t,\xi)-h(t,\eta)|^2 \le \tau(\|\xi-\eta\|^2),$$

where $\tau(\bullet)$ is a concave, nondecreasing, and continuous function from R^+ to R^+ such that $\tau(0)=0, \tau(u)>0$ for $u>0$ and $\int_{0^+}\frac{du}{\tau(u)}=\infty.$

(H2) For all $t_0 \le t \le T$, it follows that f(t,0), σ(t,0), h(t,0)∈L^2 such that

$$|f(t,0)|^2 \vee |\sigma(t,0)|^2 \vee |h(t,0)|^2 \le C_0,$$

where $C_0>0$ is a constant.

(H3) There exists a positive number K∈(0,1) such that, for all $\xi,\eta \in C((-\infty,0];R^d)$ and $t_0 \le t \le T$,

$$|g(\xi)-g(\eta)| \quad \le K\|\xi-\eta\|.$$

Remark 2.1 To demonstrate the generality of our results, let us illustrate it using concrete function $\tau(\bullet)$.
Let ε∈(0,1). Set

$$\tau_1(u)=u, \ \forall u \ge 0.$$

$$\tau_2(u)=\begin{cases} u\log(\frac{1}{u}), & 0\le u\le \varepsilon, \\ \varepsilon\log(\frac{1}{\varepsilon})+\tau_2'(\varepsilon-)(u-\varepsilon), & u>\varepsilon, \end{cases}$$

$$\tau_3(u)=\begin{cases} u\log(\frac{1}{u})\log\log(\frac{1}{u}), & 0\le u\le \varepsilon, \\ \varepsilon\log(\frac{1}{\varepsilon})\log\log(\frac{1}{\varepsilon})+\tau_3'(\varepsilon-)(u-\varepsilon), & u>\varepsilon, \end{cases}$$

where ε is sufficiently small and τ_i', $i=2,3$ is the left derivative of τ_i, $i=2,3$ at the point ε. Then τ_i, $i=1,2,3$ are concave nondecreasing functions definition on R^+ satisfying $\int_{0^+}\frac{du}{\tau_i(u)}=\infty.$

Remark 2.2 If there exist a positive constant δ, such that τ(u)= δu, u∈$C((-\infty,0];R^d)$ then assumption **(H1)** implies the Lipschitz condition**.**

3. EXISTENCE AND UNIQUENESS OF SOLUTION

In this section, we shall investigate the existence and uniqueness of the solution to Eq.(1.1) with initial data (1.2).

The main result of the paper is the following theorem.

Theorem 3.1 Assume the assumptions of **(H1)**-**(H3)** hold. Then, there exist a unique solution to Eq.(1.1) with initial data (1.2). Moreover, the solution belongs to B_T.

To prove the uniqueness of the solution for Eq.(1.1) with initial data (1.2), we shall establish the following lemma.

Lemma 3.1 Let the assumptions of Theorem 3.1 hold. If x(t) is a solution of Eq.(1.1) with initial data (1.2), then there exists a positive constant C* such that

$$\|x(t)\|^2_{B_T} \le C^*.$$

Proof For every integer n ≥ 1, define the stopping time

$$\tau_n = T\wedge \inf\{t\in[t_0,T]: \|x_t\| \ge n\}.$$

Obviously, as $n\to\infty$, $\tau_n \uparrow T$ a.s. Let $x^n(t) = x(t\wedge\tau_n)$, for $-\infty<t\le T$. Then, for $t\in[t_0,T]$, $x^n(t)$ satisfy the following equation:

$$x^n(t)=\varphi(0)-g(\varphi)+g^n(x_t) + \int_{t_0}^{t} f(s,x_s^n)\mathbf{1}_{[t_0,\tau_n]}ds + \int_{t_0}^{t}\sigma(s,x_s^n)\mathbf{1}_{[t_0,\tau_n]}dB(s) + \int_{t_0}^{t} h(s,x_s^n)\mathbf{1}_{[t_0,\tau_n]}dN(s),$$

where $\mathbf{1}_A$ is the indicator function of a set A. Set

$$I^n(t):=\varphi(0)+\int_{t_0}^{t} f(s,x_s^n)\mathbf{1}_{[t_0,\tau_n]}ds + \int_{t_0}^{t}\sigma(s,x_s^n)\mathbf{1}_{[t_0,\tau_n]}dB(s) + \int_{t_0}^{t} h(s,x_s^n)\mathbf{1}_{[t_0,\tau_n]}dN(s).$$

By Lemma 2.3 (p. 204) in [1] and assumption **(H3)**, we have

$$|x^n(t)|^2 \le \frac{1}{K}|g(x_t^n)-g(\varphi)|^2+\frac{1}{1-K}|I^n(t)|^2 \le \sqrt{K}\|x_t^n\|^2+\frac{K}{1-\sqrt{K}}\|\varphi\|^2+\frac{1}{1-K}|I^n(t)|^2.$$

Hence

$$E\sup_{t_0\le s\le t}(|x^n(t)|^2) \le \sqrt{K}E\sup_{-\infty\le s\le t}(|x^n(t)|^2) + \frac{K}{1-\sqrt{K}}E\|\varphi\|^2+\frac{1}{1-K}E\sup_{t_0\le s\le t}|I^n(t)|^2 \ .$$

Noting the fact that

$\sup_{-\infty<s\le t}|x^n(s)|^2 \le \|\varphi\|^2 + \sup_{t_0\le s\le t}|x^n(s)|^2$,

we can get

$$E\sup_{-\infty\le s\le t}(|x^n(t)|^2) \le \frac{1+K-\sqrt{K}}{(1-\sqrt{K})^2}E\|\varphi\|^2 + \frac{1}{(1-\sqrt{K})(1-K)}E\sup_{t_0\le s\le t}|I^n(t)|^2.$$

Using the basic inequality $|a+b+c+d|^2 \le 4|a|^2+4|b|^2+4|c|^2+4|d|^2$, Hölder's inequality, and for the jump integral, we convert to the compensated Poisson process $\tilde{N}(t) := N(t)-\lambda t$, which is a martingale, we have

$$|I^n(t)|^2 \le 4|\varphi(0)|^2 + 4\left|\int_{t_0}^{t} f(s,x_s^n)\mathbf{1}_{[t_0,\tau_n]}ds\right|^2$$
$$+4\left|\int_{t_0}^{t}\sigma(s,x_s^n)\mathbf{1}_{[t_0,\tau_n]}dB(s)\right|^2$$
$$+4\left|\int_{t_0}^{t}h(s,x_s^n)\mathbf{1}_{[t_0,\tau_n]}dN(s)\right|^2$$
$$\le 4|\varphi(0)|^2 + 4(t-t_0)\int_{t_0}^{t}|f(s,x_s^n)|^2ds$$
$$+4\left|\int_{t_0}^{t}\sigma(s,x_s^n)\mathbf{1}_{[t_0,\tau_n]}dB(s)\right|^2$$
$$+8\left|\int_{t_0}^{t}h(s,x_s^n)\mathbf{1}_{[t_0,\tau_n]}d\tilde{N}(s)\right|^2$$
$$+8\lambda^2(t-t_0)\int_{t_0}^{t}|h(s,x_s^n)|^2ds.$$

By Theorem 7.2 (p. 40) in [1], the Doob martingale inequality (apply for the jump integral, see, for example [1]), assumptions (**H1**) and (**H2**), one can show that

$$E\sup_{t_0\le s\le t}|I^n(t)|^2 \le 4E|\varphi(0)|^2 + 4(t-t_0)$$
$$\times E\int_{t_0}^{t}|f(s,x_s^n)|^2ds + 16E\int_{t_0}^{t}|\sigma(s,x_s^n)|^2ds$$
$$+8[4\lambda+\lambda^2(t-t_0)]E\int_{t_0}^{t}|h(s,x_s^n)|^2ds$$
$$\le 4E\|\varphi\|^2 + 8C_0(T-t_0)[4+(T-t_0)$$
$$+2(4\lambda+\lambda^2(T-t_0))] + 8[4+(T-t_0)$$
$$+2(4\lambda+\lambda^2(T-t_0))]E\int_{t_0}^{t}\tau(\|x_s^n\|^2)ds.$$

Given that $\tau(\bullet)$ is concave and $\tau(0)=0$, we can find a pair of positive constants a and b such that $\tau(u)\le a+bu$, for all $u\ge 0$. So, we obtain that

$$E\sup_{t_0\le s\le t}|I^n(t)|^2 \le 4E\|\varphi\|^2 + 8(C_0+a)(T-t_0)\times$$
$$[4+(T-t_0)+2(4\lambda+\lambda^2(T-t_0))] +$$
$$8b[4+(T-t_0)+2(4\lambda+\lambda^2(T-t_0))]\times$$
$$\int_{t_0}^{t}E(\|x_s^n\|^2)ds \le C_1 + C_2\int_{t_0}^{t}E\sup_{-\infty\le r\le s}(|x^n(r)|^2)ds,$$

where

$$C_1 := 4E\|\varphi\|^2 + 8(C_0+a)(T-t_0)\times [4+(T-t_0)+2(4\lambda+\lambda^2(T-t_0))]$$

and

$$C_2 := 8b[4+(T-t_0)+2(4\lambda+\lambda^2(T-t_0))].$$

Thus, we can get

$$E\sup_{-\infty\le s\le t}(|x^n(t)|^2) \le \frac{C_1}{(1-\sqrt{K})(1-K)} + \frac{1+K-\sqrt{K}}{(1-\sqrt{K})^2}E\|\varphi\|^2 + \frac{C_2}{(1-\sqrt{K})(1-K)}\int_{t_0}^{t}E\sup_{-\infty\le r\le s}(|x^n(r)|^2)ds.$$

By the Gronwall inequality yields that

$$E\sup_{-\infty\le s\le t}(|x^n(t)|^2) \le C_3e^{C_4},$$

where

$$C_3 := \frac{C_1}{(1-\sqrt{K})(1-K)} + \frac{1+K-\sqrt{K}}{(1-\sqrt{K})^2}E\|\varphi\|^2$$

and

$$C_4 := \frac{C_2}{(1-\sqrt{K})(1-K)}.$$

Letting t = T, it then follows that

$$E\sup_{-\infty\le s\le T}(|x\ (s\wedge\tau_n)|^2) \le C_3e^{C_4}.$$

Consequently,

$$E\sup_{-\infty\le s\le \tau_n}(|x(s)|^2)\le C_3e^{C_4}.$$

Letting $n\to\infty$, it then implies the following inequality

$$E\sup_{-\infty\le s\le T}(|x(s)|^2)\le C_3e^{C_4}.$$

Thus, the desired result holds with $C^*:=C_3e^{C_4}$. This completes the proof of Lemma 3.1.

Now, motivated by [10], we shall introduce the sequence of successive approximations to Eq.(2.1) as follows:

Define $x^0(t)=\varphi(0)$ for all $t_0\le t\le T$. Let $x^n_{t_0}=\varphi$, $-\infty<t\le 0$, n=0,1,2, … and for all $t_0\le t\le T$, n=1,2, …, we define the following iterative scheme:

$$x^n(t)=\varphi(0)-g(\varphi)+g(x^n_t)+\int_{t_0}^{t}f(s,x^{n-1}_s)ds$$

$$+\int_{t_0}^{t}\sigma(s,x^{n-1}_s)dB(s)+\int_{t_0}^{t}h(s,x^{n-1}_s)dN(s).\quad(3.1)$$

Next, we prove the main result of our paper.

Proof of Theorem 3.1. The proof is split into the following three steps.

Step 1. We claim that the sequence $\{x^n_t\}_{n\ge0}$ is bounded. Obviously, $x^0(t)\in B_T$. Moreover, by the same way as in the proof of Lemma 3.1 and note that

$$\max_{1\le n\le k}E\sup_{-\infty<s\le t}|x^{n-1}(s)|^2\le E\|\varphi\|^2+$$

$$\max_{1\le n\le k}E\sup_{-\infty<s\le t}|x^n(s)|^2,$$

we can easily show that $x^n(t)\in B_T$, for , $-\infty<t\le T$ and n=1,2,.... This proves the boundedness of $\{x^n_t\}_{n\ge0}$.

Step 2. We claim that the sequence $\{x^n_t\}_{n\ge0}$ is a Cauchy sequence in B_T. For m, n ≥ 0 and $t\in[t_0,T]$, from (3.1) and by Lemma 2.3 in [1], we can get

$$|x^n(t)-x^m(t)|^2\le\frac{1}{K}|g(x^n_t)-g(x^m_t)|^2$$

$$+\frac{3}{1-K}\left|\int_{t_0}^{t}[f(s,x^{n-1}_s)-f(s,x^{m-1}_s)]ds\right|^2$$

$$+\frac{3}{1-K}\left|\int_{t_0}^{t}[\sigma(s,x^{n-1}_s)-\sigma(s,x^{m-1}_s)]dB(s)\right|^2$$

$$+\frac{3}{1-K}\left|\int_{t_0}^{t}[h(s,x^{n-1}_s)-h(s,x^{m-1}_s)]dN(s)\right|^2.$$

By Hölder's inequality, Theorem 7.2 in [1], the Doob martingale inequality for the jump integral, and assumptions (**H1**), (**H3**), we obtain that

$$E\sup_{t_0\le s\le t}(|x^n(t)-x^m(t)|^2)$$

$$\le K\,E\sup_{t_0\le s\le t}(|x^n(t)-x^m(t)|^2)$$

$$+\frac{C_5}{(1-K)}\int_{t_0}^{t}\tau\Big(E\sup_{t_0\le r\le s}(|x^{n-1}(r)-x^{m-1}(r)|^2)\Big)ds,$$

where $C_5:=3[4+8\lambda+(2\lambda^2+1)(T-t_0)]$, which further implies that

$$E\sup_{t_0\le s\le t}(|x^n(t)-x^m(t)|^2)\le\frac{C_5}{(1-K)^2}$$

$$\times\int_{t_0}^{t}\tau\Big(E\sup_{t_0\le r\le s}(|x^{n-1}(r)-x^{m-1}(r)|^2)\Big)ds.\quad(3.2)$$

Let

$$y(t):=\lim_{n,m\to+\infty}\sup E\sup_{t_0\le s\le t}(|x^n(t)-x^m(t)|^2).$$

From (3.2), for any ε > 0, we have

$$y(t)\le\varepsilon+\frac{C_5}{(1-K)^2}\int_{t_0}^{t}\tau(y(s))\,ds.$$

By the Bihari inequality [11], which implies that, for all sufficiently small ε > 0,

$$y(t)\le G^{-1}\left[G(\varepsilon)+\frac{C_5}{(1-K)^2}(T-t_0)\right],\qquad(3.3)$$

where $G(r)=\int_1^r\frac{du}{\tau(u)}$ on r > 0 and $G^{-1}(\bullet)$ is the inverse function of $G(\bullet)$. By assumption, $\int_{0^+}\frac{du}{\tau(u)}=\infty$ and the definition of $\tau(\bullet)$, one sees that $\lim_{\varepsilon\to0}G(\varepsilon)=-\infty$ and then

$$\lim_{\varepsilon\to0}G^{-1}\left[G(\varepsilon)+\frac{C_5}{(1-K)^2}(T-t_0)\right]=0.$$

Therefore, letting ε→0 in (3.3), we infer that for all $s\in[t_0,T]$, $y(t)=0$.

This shows that sequence $\{x^n_t\}_{n\ge0}$ is a Cauchy sequence in L^2. Hence, $x^n(t)\xrightarrow{L^2}x(t), n\to\infty$, that is $E|x^n(t)-x(t)|^2\xrightarrow{n\to\infty}0$. Furthermore, by the boundedness of $\{x^n_t\}_{n\ge0}$ in Step 1, letting n→∞ we can easily derive that $\|x(t)\|^2_{B_T}\le C^*$, where C* is a positive constant. This shows that $x(t)\in B_T$.

Step 3. We claim the existence and uniqueness of the solution to Eq.(1.1).

Existence: By the same way as in Step 2, and the sequence $x^n(t)$ is uniformly converge on $(-\infty,T]$, letting $n\to\infty$ in (3.1), we can derive the solution of Eq.(1.1) with the initial data (1.2). Thus the existence of the Theorem 3.1 is complete.

Uniqueness: Let both x(t) and z(t) be two solutions of Eq.(1.1). By Lemma 3.1, $x(t), z(t)\in B_T$. On the other hand, by the same way as in Step 2, we can show that there exist a positive constant C_6 such that

$$E\sup_{t_0\le s\le t}(|x(s)-z(s)|^2)$$
$$\le C_6\int_{t_0}^{t}\tau\Big(E\sup_{t_0\le r\le s}(|x(r)-z(r)|^2)\Big)ds.$$

Consequently, for any $\varepsilon > 0$

$$E\sup_{t_0\le s\le t}(|x(s)-z(s)|^2)$$
$$\le \varepsilon + C_6\int_{t_0}^{t}\tau\Big(E\sup_{t_0\le r\le s}(|x(r)-z(r)|^2)\Big)ds.$$

By Bihari's inequality, for all sufficiently small $\varepsilon > 0$, we can show that

$$E\sup_{t_0\le s\le t}(|x(s)-z(s)|^2)\le G^{-1}[G(\varepsilon)+C_6(T-t_0)],$$

where $G(r)=\int_1^r\frac{du}{\tau(u)}$ on $r > 0$ and $G^{-1}(\bullet)$ is the inverse function of $G(\bullet)$. By assumption, we get

$$E\sup_{t_0\le s\le t}(|x(s)-z(s)|^2)=0,$$

which further implies $x(s) \equiv z(s)$ almost surely for any $s\in[t_0,T]$. Therefore, for all $-\infty<s\le T$, $x(s) \equiv z(s)$ almost surely. The proof for Theorem 3.1 is thus complete.

Remark 3.1. By using methods similar to many articles about the theorems of the existence and uniqueness of the solution for SFDEs (see [1] or [9], Theorem 3.6), if non-Lipschitz condition is replaced by the local non-Lipschitz condition, then the existence and uniqueness theorem for neutral SFDEs with infinite delay and Poisson jumps in the phase space $C((-\infty,0];R^d)$ under local non-Lipschitz condition and the conditions (**H2**), (**H3**) is also derived.

Remark 3.2. If the phase space $C((-\infty,0];R^d)$ is replaced by the phase space $B((-\infty,0];R^d)$ (see [5]) which has origin was introduced by Hale and Kato [12] then by using method in our paper, conclusions of Theorem 3.1 also easily obtained.

Remark 3.3. If Eq.(1.1) with initial data (1.2) in the phase space $C((-\infty,0];R^d)$ under the non-Lipschitz condition and the weakened linear growth condition is replaced by the phase $C((-r,0];R^d)$ (i.e. with finite delay) under the uniform Lipschitz condition and the linear growth condition then Theorem 3.2 in Tan el at. [9] can be obtained by Theorem 3.1.

Remark 3.4. We have known that in paper [8], the proofs of the assertions are based on some function inequalities. If using our proof then the conclusions in paper [8] can be also obtained and we have saw that the procedures in our paper have become simpler than the procedures used in [8].

Remark 3.5. In real world problems, impulsive effects also exist in addition to stochastic effects. The theory of impulsive differential equations is much richer than the corresponding theory of differential equations without impulse effects. Differential equation with impulsive conditions constitute an important field of research due to their numerous applications in ecology, medicine biology, electrical engineering, and other areas of science. There has been a significant development in impulsive theory especially in the area of impulsive differential equation with fixed moments, see for instance the monograph by Lakshmikantham et al. [13]. Recently, the existence and uniqueness of the solution for impulsive SFDEs without infinite delay and Poisson jumps have been discussed in [14]. Therefore, it is necessary and important to consider the existence and uniqueness of the solution for SFDEs with infinite delay, Poisson jumps and impulsive effects. The results in this paper can be extended to study the existence and uniqueness of the solution for SFDEs with infinite delay, Poisson jumps and impulsive effects by employing the idea and technique as in Theorem 3.1.

4. CONCLUSION

In this paper, we have discussed for a class of neutral SFDEs with infinite delay and Poisson jumps in the phase space $C((-\infty,0];R^d)$ under non-Lipschitz condition with Lipschitz condition being considered as a special case and a weakened linear growth condition on the coefficients by means of the successive approximation. By using a iterative scheme, Bihari's inequality combined with theories of stochastic analytic, then the existence and uniqueness theorem for neutral SFDEs with infinite delay and Poisson jumps is obtained. The results in our paper extend and improve the corresponding ones announced by Xu and Hu [5], Cui and Yan [8], Tan el al. [9], Chen [10], and some other results.

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