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On pgrα Homeomorphisms in Intuitionistic Fuzzy Topological Spaces

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Abstract: In this paper, we introduce intuitionistic fuzzy pgrα closed mapping, intuitionistic fuzzy pgrα open mapping, intuitionistic fuzzy pgrα homeomorphisms and study some of their properties in intuitionistic fuzzy topological spaces.

Keywords and Phrases: Intuitionistic fuzzy topology, intuitionistic fuzzy set, intuitionistic fuzzy pgrα continuous, intuitionistic fuzzy pgrα open mapping, intuitionistic fuzzy pgrα closed mapping and intuitionistic fuzzy pgrα homeomorphisms.

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1.INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [16] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets by using the notation of fuzzy sets. On the other hand Coker [2] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper, we introduce intuitionistic fuzzy pgrα-closed mapping, intuitionistic fuzzy pgrα open mapping, intuitionistic fuzzy pgrα homeomorphisms and study some of their properties.

2. PRELIMINARIES

Throughout this paper, (X,τ), (Y,σ) and (Z,γ) (or simply X, Y and Z) denote the intuitionistic fuzzy topological spaces (IFTS for short) on which no separation axioms are assumed unless otherwise explicitly mentioned. For a subset A of X, the closure, the interior and the complement of A are denoted by cl(A), int(A) and A^c respectively. We recall some basic definitions that are used in the sequal.

2.1. Definition [1]

Let X be a nonempty set. An intuitionistic fuzzy set (IFS for short) A in X is an object having the form A= $\langle x, \mu_A, v_A; x\epsilon X\rangle$ where the functions μ_A: X→ [0,1] and v_A : X → [0,1] denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $v_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + v_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

2.2. Definition [1]

Let A and B be IFSs of the form A = $\{\langle x, \mu_A(x), v_A(x)\rangle : x \in X\}$ and B= $\{\langle x, \mu_B(x), v_B(x)\rangle : x \in X\}$. Then

1. A⊆B if and only if $\mu_A(x) \leq \mu_B(x)$ and $v_A(x) \geq v_B(x)$ for all $x \in X$,
2. A=B if and only if A⊆B and B⊆ A,
3. A^c=$\{\langle x, v_A(x), \mu_A(x)\rangle : x \in X\}$,
4. A⋂B=$\{\langle x, \mu_A(x) \wedge \mu_B(x), v_A(x) \vee v_B(x)\rangle : x \in X\}$,
5. A∪B=$\{\langle x, \mu_A(x) \vee \mu_B(x), v_A(x) \wedge v_B(x)\rangle : x \in X\}$.

For the sake of simplicity, we shall use the notation A= $\langle x, \mu_{A,} v_A\rangle$ instead of A= $\{\langle x, \mu_A(x), v_A(x)\rangle ; x \in X\}$. The intuitionistic fuzzy sets $0_{\sim}$= $\{\langle x, 0,1\rangle : x \in X\}$ and $1_{\sim}$= $\{\langle x, 1,0\rangle : x \in X\}$ are respectively the empty set and the whole set of X.

2.3. Definition [2]

An intuitionistic fuzzy topology (IFT for short) on X is a family τ of IFSs in X satisfying the following axioms.

1. $0_{\sim}$, $1_{\sim} \epsilon\ \tau$,
2. $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \epsilon\ \tau$,
3. $\cup\, G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X,τ) is called an intuitionistic fuzzy topological space (IFTS for short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X. The complement A^c of an IFOS A in an IFTS (X,τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X.

2.4. Definition [2]

Let (X,τ) be an IFTS and A=$\langle x, \mu_{A,} v_A\rangle$ be an IFS in X. Then

1. int(A) = $\cup \{G : G$ is an IFOS in X and G $\subseteq A\}$,
2. cl(A) = $\cap$ {K: K is an IFCS in X and A $\subseteq K\}$.

For any IFS A in (X,τ), we have cl (A^c) = $(\text{int}(A))^c$ and int(A^c) = $(\text{cl}(A))^c$

2.5. Definition [3]

An IFS A=$\langle x, \mu_{A,} v_A\rangle$in an IFTS (X,$\tau$) is said to be an

1. intuitionistic fuzzy semi closed set (IFSCS for short) if int(cl(A)) $\subseteq A$,
2. intuitionistic fuzzy semi open set (IFSOS for short) if $A \subseteq$ cl(int((A))
3. intuitionistic fuzzy pre closed set (IFPCS for short) if cl(int((A)) $\subseteq A$,

4. intuitionistic fuzzy pre open set (IFPOS for short) if $A \subseteq$ int(cl(A)),
5. intuitionistic fuzzy regular closed set (IFRCS for short) if cl(int(A))= A,
6. intuitionistic fuzzy regular open set (IFROS for short) if A= int(cl(A)),
7. intuitionistic fuzzy α closed set (IFαCS for short) if cl(int(cl(A))) $\subseteq A$,
8. intuitionistic fuzzy α open set (IFαOS for short) if A $\subseteq$ int(cl(int(A))).

2.6. Definition

Let A=$\langle x, \mu_{A,} \nu_A \rangle$ be an IFS in an IFTS (X,τ). Then
1. αint(A) = $\cup \{G: G$ is an IFαOS in X and G$\subseteq A\}$ and αcl(A) = $\cap$ {K: K is an IFαCS in X and A$\subseteq K\}$**[9]**,
2. $p$int(A = $\cup \{G: G$ is an IFPOS in X and G$\subseteq A\}$ and $p$cl(A) = $\cap$ {K: K is an IFPCS in X and A$\subseteq K\}$**[3]**,
3. sint(A) = $\cup \{G: G$ is an IFSOS in X and G$\subseteq A\}$ and $s$cl(A) = $\cap$ {K: K is an IFSCS in X and A$\subseteq K\}$**[9]**.

2.7. Definition [15]

An IFS A of an IFTS (X,τ) is called an intuitionistic fuzzy regular α open set ((IFRαOS for short) if there exist an IFROS U such that U $\subseteq A \subseteq \alpha$cl(U).

2.8. Definition

An IFS A=$\langle x, \mu_{A,} \nu_A \rangle$ in an IFTS (X,τ) is called an

1. intuitionistic fuzzy generalized closed set (IFGCS for short) if cl(A) $\subseteq U$, whenever A$\subseteq U$ and U is an IFOS in X **[13]**,
2. intuitionistic fuzzy α generalized closed set (IFαGCS for short) if αcl(A) $\subseteq U$, whenever A$\subseteq U$ and U is an IFOS in X **[9]**,
3. intuitionistic fuzzy weakly generalized closed set (IFWGCS for short) if cl(int(A)) $\subseteq U$, whenever A$\subseteq U$ and U is an IFOS in X **[7]**,
4. intuitionistic fuzzy generalized semi closed set (IFGSCS for short) if scl(A) $\subseteq U$,whenever A$\subseteq U$ and U is an IFOS in X **[11]**,
5. intuitionistic fuzzy regular generalized α closed set (IFRGαCS for short) if αcl(A) $\subseteq U$, whenever A$\subseteq U$ and U is an IFRαOS in X **[5]**.

An IFS A is said to be an intuitionistic fuzzy generalized open set (IFGOS for short), intuitionistic fuzzy α generalized open set (IFαGOS for short), intuitionistic fuzzy weakly generalized open set (IFWGOS for short), intuitionistic fuzzy generalized semi open set (IFGSOS for short) and intuitionistic fuzzy regular generalized α open set (IFRGαOS for short) if the complement of A is an IFGCS, IFαGCS, IFWGCS, IFGSCS, and IFRGαCS respectively

2.9. Definition [6]

An IFS A = $\langle x, \mu_{A,} \nu_A \rangle$ in an IFTS (X,τ) is called an intuitionistic fuzzy regular weakly generalized closed set (IFRWGCS for short) if cl(int(A)) $\subseteq U$, whenever A $\subseteq$ U and U is an IFROS in X. An IFS A is called an intuitionistic fuzzy regular weakly generalized open set (IFRWGOS for short) in X if the complement of A is an IFRWGCS in X.

2.10. Definition [14]

An IFS A in an IFTS (X,τ) is said to be an intuitionistic fuzzy pgrα closed set (IF pgrα CS for short) if pcl (A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFRαOS in X. The family of all IF pgrαCSs of an IFTS (X, τ) is denote by IFpgrαC(X).

2.11. Definition [3]

A mapping f: (X,τ) $\rightarrow$ (Y,σ) is called an intuitionistic fuzzy continuous mapping (IF continuous mapping for short) if f^{-1}(B) is an IFOS in (X,τ) for every IFOS B of (Y,σ).

2.12. Definition [4]

Let f: (X,τ) $\rightarrow$ (Y,σ) be a mapping. Then f is said to be an
1. intuitionistic fuzzy α continuous mapping (IF α continuous mapping for short) if f^{-1}(B) ϵ IFαO(X) for every B$\epsilon\sigma$,
2. intuitionistic fuzzy pre continuous mapping (IFP continuous mapping for short) if f^{-1}(B) ϵ IFPO(X) for every B ϵ σ.

2.13. Defintion [5]

Let f: (X,τ) $\rightarrow$(Y,σ) be a mapping. Then f is said to be an intuitionistic fuzzy regular generalized α continuous mapping (IFRGα continuous mapping for short) if f^{-1}(B) is an IFRGαCS in (X,τ) for every IFCS B of (Y,σ).

2.14. Definition [8]

Let f: (X,τ) $\rightarrow$(Y,σ) be a mapping. Then f is said to be an intuitionistic fuzzy regular weakly generalized continuous mapping (IFRWG continuous mapping for short) if f^{-1}(B) is an IFRWGCS in (X,τ) for every IFCS B of (Y,σ).

2.15. Definition [14]

A mapping f: (X,τ) $\rightarrow$(Y,σ) is called an intuitionistic fuzzy pgrα continuous (IFpgrα continuous for short) mapping if f^{-1}(V) is an IFpgrαCS in (X,τ) for every IFCS V of (Y,σ).

2.16. Definition [12]

A mapping f: (X,τ) $\rightarrow$(Y,σ) is called an intuitionistic fuzzy closed mapping (IFCM for short) if f(A) is an IFCS in Y for each IFCS A in X.

2.17. Definition [12]

A mapping f: (X,τ) $\rightarrow$(Y,σ) is called an
1. intuitionistic fuzzy semi open mapping (IFSOM for short) if f(A) is an IFSOS in Y for each IFOS A in X.
2. intuitionistic fuzzy pre open mapping (IFPOM for short) if f(A) is an IFPOS in Y for each IFOS A in X.

2.18. Definition [5]

A mapping f: (X,τ) $\rightarrow$(Y,σ) is an intuitionistic fuzzy rgα closed mapping (IFRGα CM for short) if image of every IFCS of X is an IFRGα CS in Y.

2.19. Definition [2]

Let f: (X,τ) →(Y,σ) be a mapping. Then f is said to be intuitionistic fuzzy homeomorphism (IF homeomorphism for short) if f and f^{-1} are IF continuous mappings.

2.20. Definition [10]

Let f: (X,τ) →(Y,σ) be a mapping. Then f is said to be intuitionistic fuzzy α homeomorphism (IF α homeomorphism for short) if f and f^{-1} are IF α continuous mappings.

3. INTUITIONISTIC FUZZY pgrα CLOSED MAPPING AND INTUITIONISTIC FUZZY pgrα OPEN MAPPING

In this section we introduce intuitionistic fuzzy pgrα closed mapping, intuitionistic fuzzy pgr α open mapping and investigate some of its properties.

3.1. Definition

A mapping f: (X,τ) →(Y,σ) is called an intuitionistic fuzzy pgrα closed mapping (IF pgrα CM for short) if f(A) is an IF pgrα CS in Y for each IFCS A in X.

3.2. Example

Let X={a,b}, Y={u,v} and G_1=<x,(0.8,0.7),(0.2,0.2)>, G_2=<x,(0.3,0.3),(0.7,0.7)>. Then τ= {$0_\sim$, G_1, $1_\sim$} and σ ={$0_\sim$,G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a)=u and f(b)=v. Then f is an IF pgrα CM.

3.3. Definition

A mapping f: (X,τ) →(Y,σ) is called an intuitionistic fuzzy pgrα open mapping (IF pgrα OM for short) if f(A) is an IF pgrα OS in Y for each IFOS A in X.

3.4. Definition

A mapping f: (X,τ) →(Y,σ) is called an intuitionistic fuzzy ipgrα closed mapping (IFipgrα CM for short) if f(A) is an IF pgrα CS in Y for each IF pgrα CS A in X.

3.5. Example

Let X={a,b}, Y={u,v} and G_1=<x,(0.3,0.3),(0.7,0.7)>, G_2=<x,(0.8,0.7),(0.2,0.2)>. Then τ= {$0_\sim$, G_1, $1_\sim$} and σ ={$0_\sim$,G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a)=u and f(b)=v. Then f is an IFipgrα CM.

3.6. Definition

A mapping f: (X,τ) →(Y,σ) is called an intuitionistic fuzzy ipgrα open mapping (IFipgrα OM for short) if f(A) is an IF pgrα OS in Y for each IF pgrα OS A in X.

3.7. Definition

Let (X,τ) be an IFTS and A= $\langle x,\mu_{A,}\nu_A\rangle$ be an IFS in X. Then pgrα -interior of A (pgrαint(A) for short) and pgrα -closure of A (pgrαcl(A) for short) are defined as

1. pgrαint(A) = ∪ {G: G is an IF pgrα OS in X and G ⊆ A},

2. pgrαcl(A) = ∩ {K: K is an IF pgrα CS in X and A ⊆ K}.

3.8. Theorem

Every IFCM is an IF pgrα CM but not conversely.

Proof:

Let f: (X, τ) → (Y, σ) be an IF closed mapping. Let A be an IFCS in X. Then f(A) is IFCS in Y. This implies that f(A) is an IF pgrα CS inY. Hence f is an IF pgrα closed mapping.

3.9. Example

In Example 3.2, f: (X, τ) → (Y, σ) is an IF pgrα CM but not an IFCM.

3.10. Theorem

Every IF α CM is an IF pgrα CM but not conversely.

Proof:

Let f: (X, τ) → (Y, σ) be an IFα closed mapping. Let A be an IFCS in X. Then f(A) is IF α CS in Y. This implies that f(A) is an IF pgrα CS in Y. Hence f is an IF pgrα closed mapping.

3.11. Example

In Example 3.2, f: (X, τ) → (Y, σ) is an IF pgrα CM but not an IF α CM.

3.12. Theorem

Every IFPCM is an IF pgrα CM but not conversely.

Proof:

Let f: (X, τ) → (Y, σ) be an IFP closed mapping. Let A be an IFCS in X. Then f(A) is IFPCS in Y. This implies that f(A) is an IF pgrα CS in Y. Hence f is an IF pgrα closed mapping.

3.13. Example

Let X={a,b}, Y={u,v} and G_1=<x,(0.3,0.1),(0.7,0.9)>, G_2=<x,(0.7,0.9),(0.3,0.1)>. Then τ= {$0_\sim$, G_1, $1_\sim$} and σ ={$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a)=u and f(b)=v. Then f is an IF pgrα CM but not IFPCM.

3.14. Theorem

Every IFRG α CM is an IF pgrα CM but not conversely.

Proof:

Let f: (X, τ) → (Y, σ) be an IFRG α CM. Let A be an IFCS in X. Then f(A) is IFRG α CS in Y. This implies that f(A) is an IF pgrα CS in Y. Hence f is an IF pgrα closed mapping.

3.15. Example
In Example 3.2, f: (X, τ) → (Y, σ) is an IF pgrα CM but not an IFRG α CM.

3.16. Theorem
Every IF ipgrα CM is an IF pgrα CM but not conversely.

Proof:
Let f: (X, τ) → (Y, σ) be an IF ipgrα CM. Let A be an IFCS in X. Then A is IF pgrα CS in Y. This implies that f(A) is an IF pgrα CS in Y. Hence f is an IF pgrα closed mapping.

3.17. Example
Let X={a,b}, Y={u,v} and G_1=<x,(0.8,0.7),(0.2,0.2)>, G_2=<x,(0.3,0.3),(0.7,0.7)>. Then τ= {$0_\sim$, G_1, $1_\sim$} and σ ={$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a)=u and f(b)=v. Then f is an IF pgrα CM but not IF ipgrα CM.

3.18. Theorem
A mapping f: (X, τ) → (Y, σ) is IF pgrα closed mapping if and only if for each subset S of Y and for each IFOS U containing f^{-1}(S) there is an IF pgrα OS V of Y such that S⊆V and f^{-1}(V) ⊆ U.

Proof:
Suppose f is an IF pgrα closed. Let S be a subset of Y and U is an IFOS of X such that f^{-1}(S) ⊆ U. Then V = Y-f(X-U) is an IF pgrα OS containing S such that f^{-1}(V) ⊆ U.

Conversely, suppose that F is an IFCS in X. Then f^{-1}(Y-f(F)) ⊆ X-F, X-F is an IFOS in X. By hypothesis, there is an IF pgrα OS V of Y such that Y-f(F) ⊆ V and f^{-1}(V) ⊆ X-F.Therefore F ⊆ X-f^{-1}(V). Hence Y-V⊆ f(F) ⊆ f(X-f^{-1}(V)) ⊆ Y-V, which impiles f(F)=Y-V. Since Y-V is an IF pgrα CS in Y, f(F) is an IF pgrα CS in Y and therefore f is an IF pgrα closed mapping.

3.19. Theorem
If f: (X, τ) → (Y, σ) is an IF pgrα CM and A is an IFCS of X, then f_A:A→Y is IF pgrα CM.

Proof:
Let B⊆A be ab IFCS in A, then B is an IFCS in X. Since A is an IFCS in X, f(B) is an IF pgrα CS in Y as f is IF pgrα CM. But f(B)= f_A(B). So f_A(B) is an IF pgrα CS in Y. Therefore f_A is an IF pgrα CM.

3.20. Remark

Composition of two IF pgrα CMs need not be an IF pgrα CM.

3.21. Example
Let X={a,b}, Y={c,d} and Z={u,v}. Let G_1=<x,(0.5,0.6),(0.5,0.4)>, G_2=<x,(0.6,0.1),(0.4,0.3)> and G_3=<x,(0.4,0.4),(0.6,0.6)>,. Then τ= {$0_\sim$, G_1, $1_\sim$}, σ ={$0_\sim$, G_2, $1_\sim$} and γ ={$0_\sim$, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a)=c and f(b)=d and g: (Y, σ) → (Z,γ) by g(c)=u and g(d)=v. Then f and g are an IF pgrα CM. But their composition gof: (X, τ) → (Z,γ) need not be an IF pgrα CM.

3.22. Theorem
If f: (X, τ) → (Y, σ) is an IFCM and g: (Y, σ) → (Z,γ) is an IF pgrα CM, then gof: (X, τ) → (Z,γ) is an IF pgrα CM.

Proof:
Let H be an IFCS in X. Then f(H) is an IFCS. But (gof)(H)=g(f(H)) is an IF pgrα CS as g ia an IF pgrα CM. Thus gof is an IF pgrα CM.

3.23. Theorem
If f: (X, τ) → (Y, σ) is a bijective mapping, then the following statements are equivalent

1. f is an IF pgrα OM.

2. f ia an IF pgrα CM.

3. f^{-1}: (Y, σ) →(X, τ) is an IF pgrα continuous.

Proof:
(1)⇒(2) Let U be an IFCS in X and f be an IF pgrα OM. Then X-U is an IFOS in X. By hypothesis, we get f(X-U) is an IF pgrα OS in Y. That is Y-f(X-U)-f(U) is an IF pgrα in Y.

(2) ⇒(3) L is an et U ne an IFCS in X. By assumption, f(U) IF pgrα CS in Y. As f(U)=$(f^{-1})^{-1}$(U), f^{-1} is an IF pgrα continuous.

(3) ⇒ (1) Let U be an IFOS in X. By assumption $(f^{-1})^{-1}$(U) =f(U). That is f(U) is an IF pgrα OS in Y. Hence f is an IF pgrα OM.

3.24. Definition
A space (X, τ) is called an IFpgr$\alpha T_{1/2}$ space if every IF pgrα CS is an IFα CS.

3.25. Definition
A space (X, τ) is called an IFpgr$T_{1/2}$ space if every IF pgrα CS is an IFCS.

3.26. Definition
A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy pgrα irresolute (IF pgrα irresolute in short) mappings if f^{-1}(V) is an IF pgrα CS in (X, τ) for every IF pgrα CS V of (Y, σ).

3.27. Theorem
For any bijective mapping f: (X, τ) → (Y, σ), then the following are equivalent

1. f^{-1}: (Y, σ) →(X, τ) is an IF pgrα irresolute mapping
2. f is an IF ipgrα OM
3. f is an IF ipgrα CM

Proof:

(1)⇒(2) Let U be an IF pgrα OS in X. By (1), $(f^{-1})^{-1}$(U) =f(U) is an IF pgrα OS in Y So f is an IF ipgrα OM.

(2) ⇒(3) Let V be an IF pgrα CS in X. By (2), f(X-V)= Y-f(V) is an IF pgrα OS in Y. That is f(V) is an IF pgrα CS in Y and so f is an IF ipgrα CM.

(3) ⇒ (1) Let V be an IF pgrα CS in X. By (3), f(V)= $(f^{-1})^{-1}$(V) is an IF pgrα CS in Y. Hence (1) holds.

3.28. Theorem

If f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z,γ) are IF ipgrα CM, then gof: (X, τ) → (Z,γ) is an IF ipgrα CM.

Proof:

Let V be an IF pgrα CS in X. Since f is an IF ipgrα CM, f(V) is an IF pgrα CS in Y. Then g(f(V)) is an IF pgrα CS in Z. Hence gof is an IF ipgrα CM.

3.29. Theorem

If f: (X, τ) → (Y, σ) is an IF pgrα CM and g: (Y, σ) → (Z,γ) is an IF ipgrα CM, then gof: (X, τ) → (Z,γ) is an IF pgrα CM.

Proof:

Let V be an IFCS in X. Since f is an IF pgrα CM, f(V) is an IF pgrα CS in Y. Then g(f(V)) is an IF pgrα CS in Z. Hence gof is an IF pgrα CM.

4 INTUITIONISTIC FUZZY pgrα HOMEOMORPHISM

In this section, we introduce the concept of intuitoinstic fuzzy pgrα homeomorphism, intuitionistic fuzzy ipgrα homeomorphism and study some of their properties.

4.1. Definition

A bijective mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy pgr α homeomorphism (IF pgr α homeomorphism in short) if f and f^{-1} are IF pgrα continuous mapping.

4.2. Example

Let X={a,b}, Y={u,v} and G_1=<x,(0.3,0.2),(0.6,0.7)>, G_2=<x,(0.8,0.9),(0.2,0.1)>. Then τ= {$0_\sim$, G_1, $1_\sim$} and σ ={$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a)=u and f(b)=v. Then f is an IF pgrα continuous mapping and f^{-1} is also an IF pgrα continuous mapping. Therefore f is an IF pgr α homeomorphism.

4.3. Theorem

Every IF homeomorphism is an IF pgrα homeomorphism but not conversely.

Proof:

Let f: (X, τ) → (Y, σ) be an IF homeomorphism. Then f and f^{-1} are IF continuous mapping. This implies that f and f^{-1} are IF pgrα continuous mapping, that is the mapping f is an IF pgrα homeomorphism.

4.4. Example

Let X={a,b}, Y={u,v} and G_1=<x,(0.2,0.3),(0.8,0.7)>, G_2=<x,(0.6,0.8),(0.3,0.2)>. Then τ= {$0_\sim$, G_1, $1_\sim$} and σ ={$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a)=u and f(b)=v. Then f is an IF pgrα continuous mapping and f^{-1} is also an IF pgrα continuous mapping. Therefore f is an IF pgr α homeomorphism but not IF homeomorphism.

4.5. Theorem

Every IF α homeomorphism is an IF pgrα homeomorphism but not conversely.

Proof:

Let f: (X, τ) → (Y, σ) be an IF α homeomorphism. Then f and f^{-1} are IF α continuous mapping. This implies that f and f^{-1} are IF pgrα continuous mapping, that is the mapping f is an IF pgrα homeomorphism.

4.6. Example

Let X={a,b}, Y={u,v} and G_1=<x,(0.5,0.1),(0.5,0.9)>, G_2=<x,(0.2,0.2),(0.7,0.8)>. Then τ= {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a)=u and f(b)=v. Therefore f is an IF pgrα homeomorphism. Consider the IFCS A= <x,(0.7,0.8),(0.2,0.2)> in Y. Then f^{-1} (A) = <x,(0.7,0.8),(0.2,0.2)> is not IF α CS in X. This implies that f is not an IF α continuous mapping. Hence f is not an IF α homeomorphism.

4.7. Theorem

Let f: (X, τ) → (Y, σ) be an IF pgrα homeomorphism, then f is an IF homeomorphism if X and Y are IFpgr$T_{1/2}$ space.

Proof:

Let B be an IFCS in Y. Then f^{-1} (B) is an IF pgrα CS in X, by hypothesis. Since X is an IFpgr$T_{1/2}$ space, f^{-1} (B) is an IFCS in X. Hence f is an IF continuous mapping. By hypothesis f^{-1}: (Y, σ) →(X, τ) is a IF pgrα continuous mapping. Let A be an IFCS in X. Then $(f^{-1})^{-1}$(A) =f(A) is an IF pgrα CS in Y, by hypothesis. Since Y is an IFpgr$T_{1/2}$ space, f(A) is an IFCS in Y. Hence f^{-1} is an IF continuous mapping. Therefore the mapping f is an IF homeomorphism.

4.8. Theorem

Let f: (X, τ) → (Y, σ) be an IF pgrα homeomorphism, then f is an IF α homeomorphism if X and Y are IFpgr$\alpha T_{1/2}$ space.

Proof:

Let B be an IFCS in Y. Then f^{-1} (B) is an IF pgrα CS in X, by hypothesis. Since X is an IFpgr$\alpha T_{1/2}$ space, f^{-1} (B) is an IF α CS in X. Hence f is an IF α continuous mapping. By hypothesis f^{-1}: (Y, σ) →(X, τ) is a IF pgrα continuous mapping. Let A be an IFCS in X. Then $(f^{-1})^{-1}$(A) =f(A) is an IF pgrα CS in Y, by hypothesis. Since Y is an IFpgr$\alpha T_{1/2}$ space, f(A) is an IF α CS in Y. Hence f^{-1} is an IF α continuous mapping. Therefore the mapping f is an IF α homeomorphism.

4.9. Theorem

Let f: (X, τ) → (Y, σ) be a bijective mapping. If f is an IF pgr α continuous mapping, then the following are equivalent

1.f is an IF pgrα CM.

2.f is an IF pgrα OM.

3.f is an IF pgrα homeomorphism.

Proof:

(1)⇒(2) Let f: (X, τ) → (Y, σ) be a bijective mapping and IF pgrα CM. This implies that f^{-1}: (Y, σ) →(X, τ) is an IF pgrα continuous mapping. That is, every IFOS in X is an IF pgrα OS in Y. Hence f^{-1} is an IF pgrα OM.

(2)⇒(3) Let f: (X, τ) → (Y, σ) be a bijective mapping and IF pgrα OM. This implies that f^{-1}: (Y, σ) →(X, τ) is an IF pgrα continuous mapping. Hence f and f^{-1} are IF pgrα continuous mapping. That is, f is an IF pgr α homeomorphism.

(3)⇒(1) Let f be an IF pgrα homeomorphism. That is, f and f^{-1} are IF pgrα continuous mappings. Since every IFCS in X is an IF pgrα CS in Y, f is an IF pgrα CM.

4.10. Definition

A bijective mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy ipgr α homeomorphism (IF ipgrα homeomorphism in short) if f and f^{-1} are IF pgrα irresolute mappings.

4.11. Theorem

Every IFipgr α homeomorphism is an IF pgrα homeomorphism but not conversely.

Proof:

Let f: (X, τ) → (Y, σ) be an IF ipgrα homeomorphism. Let B be IFCS in Y. This implies B is an IF pgrα CS in Y. By hypothesis f^{-1}(B) is an IF pgrα CS in X. Hence f is an IF pgrα continuous mapping. Similarly, we can prove f^{-1} is an IF pgrα continuous mapping. Hence f and f^{-1} are IF pgrα continuous mapping. This implies that the mapping f is an IF pgrα homeomorphism.

4.12. Example

Let X={a,b}, Y={u,v} and G_1=<x,(0.3,0.2),(0.7,0.8)>, G_2=<x,(0.9,0.9),(0.1,0.1)>. Then τ= {$0_\sim$, G_1, $1_\sim$} and σ ={$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a)=u and f(b)=v. Therefore f is an IF pgr α homeomorphism. Consider the IFCS A=<x,(0.4,0.2),(0.6,0.8)> in Y. Clearly A is an IF pgrα CS. But f^{-1}(A) is not IF α CS in X. That is f is not an IF pgrα irresolute mapping. Hence f is not an IF ipgr α homeomorphism.

4.13. Theorem

The composition of two IF ipgrα homeomorphisms is IF ipgrα homeomorphism in general.

Proof:

Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z,γ) be two any IF ipgrα homeomorphisms. Let A be an IF pgrα CS in Z. Then by hypothesis, g^{-1}(A) is an IF pgrα CS in Y. Again by hypothesis, $f^{-1}(g^{-1}$(A)) is an IF pgrα CS in X. Therefore gof is an IF pgrα irresolute mapping. Now let B be an IF pgrα CS in X. Then by hypothesis, f(B) is an IF pgrα CS in Y and also g(f(B)) is an IF pgrα CS in Z. This implies gof is an IF pgrα irresolute mapping. Hence gof is an IF ipgrα homeomorphism.

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