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An Inventory Model with Variable Demand Rate for Deteriorating Items under Permissible Delay in Payments

Ajay Kumar Agrawal
Motilal Nehru College
University Of Delhi New Delhi,
India

Kuldeep
Department Of Mathematics
Lady Shri Ram College For Women
University Of Delhi
New Delhi, India

Abstract: A continuous production control inventory model for deteriorating items with variable demand rate is developed. Demand rate is the linear function of time. In this paper we have done all work in the environment of permissible delay of payments. A number of structural properties of the inventory system are studied analytically. We have discussed the minimum total system cost under the condition of permissible delay is relaxed to that at the end of the credit period, the retailer will make a partial payment on total purchasing cost to the supplier and pay off the remaining balance by loan from the bank. Numerical examples are taken to illustrate the procedure of finding the optimal total inventory cost and production cycle time. Sensitivity analysis is carried out to demonstrate the effects of changing parameter values on the optimal solution of the system.

Keywords: Inventory, EPQ Model, Deteriorating item, variable demand rates, permissible delay in payment.

1. INTRODUCTION

The classical Economic Production Quantity (EPQ) model is widely used principally because it is so simple to use and apply. This model considers the ideal case that the value of inventory items are unaffected by time and replenishment is done instantaneously. In real life cases, however, the ideal case is not quit applicable. Inventories are often replenished periodically at certain production rate which is seldom infinite. Even for purchased items, when supply arrives at the warehouse, it may take days for receiving department to completely transfer the supply into storage room. Goods deteriorate and their value reduces with time. Electronic products may be come obsolete as technology changes: Fashion tends to depreciate the value of clothing over time; batteries die out as they age. The effect of time is even more critical for perishable goods such as food stuff and cigarettes. The effect of these two situations is that the classical inventory model has to be readjusted.

The traditional Economic Order Quantity (EOQ) model assumes that the retailer must be paid for the items as soon as the items are received. However, in practice the supplier will offer the retailer a delay period, that is the trade credit period, in paying for the amount of purchasing cost. Before the end of the trade credit period the retailer can sell the goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of trade credit period. Therefore, it is clear that the retailer will delay the payment up to the last moment of the permissible period allowed by the supplier. In the real world, the supplier often makes use to this policy to stimulate his/her customer's demand. Recently several papers have appeared in the literature that treat inventory problems with varying conditions under the consideration of permissible delay in payments. Some of the prominent papers are discussed below.

Goyal [1] established a single-item inventory model under permissible delay in payments. Chung [2] developed an alternative approach to determine the economic order quantity under condition of permissible delay in payment. Aggrawal and Jaggi [3] considered the inventory model with exponential deterioration rate under the condition of permissible delay in payments. Jamal et al [4] entended Aggarwal and Jaggi's model to shortages. There were several interesting and relevant papers related to the delay of payments such as Chu et al [5], Chung[6], Hwang and Shinn [7], Sarker et. Al, [8] Shah [9] Shinn [10] Khouja and Mehrez [11] and their references. However, these studies were developed under the assumption that the items obtained from on outside supplier and the entire lot size is delivered at the same time, when an item can be produced inhouse, the replenishment rate is also the production rate and is finite. Hence we amen Goyal's model by considering the replenishment rate is finite, the difference between purchasing price and selling cost and taking into consideration the effect of time or decay.

Specifically, the restrictive assumption of a permissible delay is related to that at the end of the credit period, the retailer will make a partial payment on total purchasing cost and pay the remaining balance by loan from the bank.

1. In reality, the demand may vary with time. Time-varying demand patterns are commonly used to reflect sales in different phases of a product life cycle in market. For example, the demand for inventory items increases over time in the growth phase and decrease in the decline phase. Donaldson [15] initially developed an inventory model with a linear trend in demand. After that many researcher's works in this environment of Goal and Aggrawal [17], Ritchie [21] Deb and Chaundhari [14]. Dave and Palel [13], Chung and Ting [12], Kishan and Mishra [19] Giri et al [16], Hwang [18], Pal and Mondal [20], have be devoted to incorporating a time-varying demand rate into their models for deteriorating items. Consequently, the main purpose of this paper is to find an optimal cycle time which minimize the total system cost. Numerical examples are also presented to illustrate the result of the proposed model.

Notation

D	Annual demand rate
P	Annual production rate
c	Unit purchasing price per item
s	Unit selling price per item of good quality
h	Unit stock holding cost per item per year excluding interest changes
Ie	Interest which can be earned per $ per year
Ip	Interest changes per $ investment in inventory per year
$I_1(t)$	The inventory level that changes with time t during production period
$I_2(t)$	The inventory level that changes with time t during non-production period
M	The trade credit period
T	The cycle time
T*	The optimal cycle time
θ	Deterioration rate of finished item
A	Setup cost per year

Assumption:

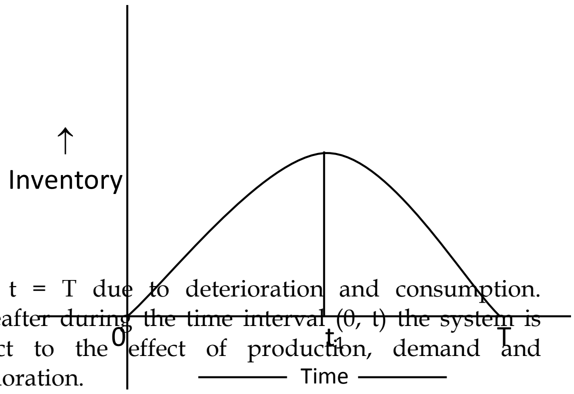
1. Production rate P is known and constant
2. Demand rate D(t) = bt, b > 0 and P > D(t) for any t
3. Shortages are not allowed
4. Time period is infinite
5. Lead time is zero
6. The constant fraction θ of on hand inventory gets deteriorated per time unit
7. s ≥ c, Ip ≥ Ie

3. MATHEMATICAL MODEL AND SOLUTION

A constant production rate starts at t = 0 and continuous up to $t = t_1$ where inventory level reaches the maximum level. Production then stops at $t = t_1$ and the inventory gradually depletes to zero at the end of the production

2. ASSUMPTIONS AND NOTATIONS

All The proposed inventory model is developed under the following assumptions and notations.



cycle t = T due to deterioration and consumption. Thereafter during the time interval (0, t) the system is subject to the effect of production, demand and deterioration.

$$\frac{dI_1(t)}{dt} = P - D(t) - \theta I_1(t) \qquad 0 \le t \le t_1$$

Or

$$\frac{dI_1(t)}{dt} = P - bt - \theta I_1(t) \qquad 0 \le t \le t_1 \quad \ldots(1)$$

With initial condition $I_1(0) = 0$

On the other hand, in the time interval (t, T), the system is affected by the combined effect of demand and deterioration. Hence, the change in the inventory level is governed by the following differential equation.

$$\frac{dI_2(t)}{dt} = -D(t) - \theta I_2(t) \qquad t_1 \le t \le T$$

Or

$$\frac{dI_2(t)}{dt} = -bt - \theta I_2(t) \qquad t_1 \le t \le T \quad \ldots..(2)$$

With ending condition $I_2(T) = 0$

The solution of the differential equation (1) and (2) are respectively represented by

$$I_1(t) = \left(\frac{P}{\theta} + \frac{b}{\theta^2}\right)\left(1 - e^{-\theta t}\right) - \frac{bt}{\theta}, \quad 0 \le t \le t_1 \quad \ldots(3)$$

And

$$I_2(t) = \frac{b}{\theta^2}\left[(\theta T - 1)e^{\theta(T-t)} - \theta t + 1\right] \qquad \ldots.(4)$$

In addition, from the boundary condition $I_1(t_1) = I_2(t_1)$ we can derive the following equation.

$$\left(\frac{P}{Q}+\frac{b}{Q^2}\right)e^{-Qt_1}=\frac{P}{Q}-\frac{b}{Q^2}(QT-1)e^{Q(T-t_1)} \qquad \ldots(5)$$

and

$$t_1=\frac{1}{\theta}\log\left[\frac{\left(P+\frac{b}{\theta}\right)+\left(bT-\frac{b}{\theta}\right)e^{\theta T}}{P}\right] \qquad \ldots(6)$$

For the moment, the individual costs are now evaluated before they are grouped together

1. Annual setup cost = $\dfrac{A}{T}$

2. Annual holding cost (excluding interest charges)

$$=\frac{h}{T}\left[\int_0^{t_1} I_1(t)dt+\int_{t_1}^{T} I_2(t)dt\right]$$

$$=\frac{h}{\theta^3 T}\left[(P\theta+b)e^{-\theta t_1}+b(\theta T-1)e^{\theta(T_1-t_1)}+\theta^2\left(Pt_1-bT^2\right)-P\theta\right]$$

4. INTEREST

Case I: when $T \le M$

(a) Interest earned: In this case, the customer sells D.T unit in total by the end of the replenishment cycle time T, and has cDT to pay the supplier in fall by the end of the credit period M, consequently, there is no interest payable. However, the interest earned per year is

$$=sI_e\left[\int_0^T D.tdt+(M-T)\int_0^T Ddt\right]/T$$

$$=\frac{1}{6}sbI_e(3M-T)T$$

(b) Interest charges

In this case no interest charges are paid for the items kept in stock.

Case II: when $M \le T$

(a) Interest earned:

During the permissible delay period, the buyer sells products and deposits the revenue into an account that earns Ie per dollar per year

Therefore, the interest earned per year is

$$=sIe\int_o^m D.t\frac{dt}{T}$$

$$=\frac{sIebM^3}{3T}$$

(b) Interest charged

The buyer sells D.M in total by the end of the permissible delay M and has cDM to pay supplier. The item in stock are charged at interest rate Ip by the supplier starting at the time M. Thereafter the buyer gradually reduces the amount of financed loan from supplier due to constant sales and revenue received. As a result the interest payable per year

$$=\frac{cIp}{T}\int_M^T I_2(t)dt$$

$$=\frac{bcIp}{T\theta^2}\left[\left(T-\frac{1}{\theta}\right)e^{\theta(T-M)}-\frac{1}{2}\theta\left(T^2-M^2\right)-M+\frac{1}{\theta}\right]$$

5. ANNUAL COST DUE TO DETERIORATED UNITS

$$=\frac{c}{T}\left[Pt_1-\int_0^{t_1}D(t)dt-\int_{t_1}^{T}D(t)dt\right]$$

$$=c\left(\frac{Pt_1}{T}-\frac{1}{2}bT\right)$$

Therefore, the total variable cost function per unit time TVC(T) is

$$TVC(T)=\begin{cases}TVC_1(T) & if\ M\le T\\ TVC_2(T) & if\ 0\le T\le M\end{cases} \qquad \ldots.(7)$$

And

TVC(T) = Setup cost + Stock holding cost + interest payable – interest earned + Annual cost due to deteriorated units

Then,

$$TVC_1(T)=\frac{A}{T}+\frac{h}{\theta^3 T}\left[(P\theta+b)e^{-\theta t_1}+b(\theta T-1)e^{\theta(T-t_1)}+\theta^2\left(pt_1-bT^2\right)-P\theta\right] \qquad \ldots(8)$$
$$+\frac{bcIp}{\theta^2 T}\left[\left(T-\frac{1}{\theta}\right)e^{\theta(T-M)}-\frac{1}{2}\theta\left(T^2-M^2\right)-M+\frac{1}{\theta}\right]-\frac{sIebM^3}{3T}+c\left(\frac{Pt_1}{T}-\frac{1}{2}bT\right)$$

$$TVC_2(T)=\frac{A}{T}+\frac{h}{\theta^3 T}\left[(P\theta+b)e^{-\theta t_1}+b(\theta T-1)e^{\theta(T-t_1)}+\theta^2\left(pt_1-bT^2\right)-P\theta\right]-\frac{1}{6}sbIe(3M-T)T \qquad \ldots(9)$$
$$+c\left(\frac{Pt_1}{T}-\frac{1}{2}bT\right)$$

Since $I_1(t_1) = I_2(t_1)$ and approximate value of $e^{\theta(T-M)}=1+\theta(T-M)+\dfrac{\theta^2(T-M)^2}{2}$ which implies equation (8) and (9) can be rearranged as following:

$$TVC_1(T)=\frac{A}{T}+\frac{h}{\theta T}\left(Pt_1-bT^2\right)+\frac{1}{2}bcIp(T-M)^2-\frac{sIebM^3}{3T}+c\left(\frac{Pt_1}{T}-\frac{1}{2}bT\right).(10)$$

And

$$TVC_2(T)=\frac{A}{T}+\frac{h}{\theta T}\left(Pt_1-bT^2\right)-\frac{1}{6}sbIe(3M-T)T+c\left(\frac{Pt_1}{T}-\frac{1}{2}bT\right).(11)$$

The objective in this paper is to find an optimal cycle time to minimize the total variable cost per unit time. For this, the optimal cycle time T_1^*, obtained by setting the derivative of equations (10) with respect to T equal to zero is the root of the following equation

i.e,

$$\frac{dTVC_1(T)}{dT}=-A+\left(\frac{C\theta+h}{\theta}\right)P\left(T\frac{dt_1}{dT}-t_1\right)-b\left(\frac{h}{\theta}+\frac{c}{2}\right)T^2+bcIp(T-M)T^2+\frac{sIebM^3}{3}=0 \quad ...(12)$$

And

$$\frac{d^2TVC_1(T)}{dT^2}=\left(\frac{c\theta+h}{\theta}\right)PT\frac{d^2t_1}{dT^2}-b\left(\frac{2h}{\theta}+c\right)T+bcIp\left(3T^2-2TM\right) \quad ...(13)$$

Where

$$\frac{dt_1}{dT}=\frac{Pb\theta Te^{\theta T}}{(P\theta+b)+b(\theta T-1)e^{\theta T}}$$

And

$$\frac{d^2t_1}{dT^2}=\frac{Pb\theta\left[\left(Pb\theta T+PQ+\theta^2T+b\right)e^{\theta T}-be^{2\theta T}\right]}{\left[(P\theta+b)+b(\theta T-1)e^{\theta T}\right]^2}$$

In this same way, the optimal cycle time T_2^*, obtained by setting the derivative of equation (11) with respect to T equal to zero is the root of the following equation

$$\frac{dTVC_2(T)}{dT}=-A+\frac{(c\theta+h)}{\theta}P\left(T\frac{dt_1}{dT}-t_1\right)-b\left(\frac{h}{\theta}+\frac{c}{2}\right)T^2-\frac{1}{6}sbIe(3M-2T)T^2 \quad ....(14)$$

And

$$\frac{d^2TVC_2(T)}{dT^2}=\frac{(c\theta+h)}{\theta}PT\frac{d^2t_1}{dT^2}-b\left(\frac{2h}{\theta}+c\right)T-bsIe(M-T)T \quad ....(15)$$

In order to find the optimal value of T_1 and T_2 i.e., T_1^* and T_2^* so that the total variable cost per unit time is minimized i.e., evaluate that values of T_1 and T_2 from equation (12) and (14) so that

$$\frac{d^2TVC_1(T)}{dT^2}>0 \text{ and } \frac{d^2TVC_2(T)}{dT^2}>0$$

6. SOLUTION PROCEDURE

The optimal solution of the system can be obtained from the following algorithm

Algorithm

Step 1: Using all the system parameters in equations (12) and (14)

Step 2: Find out all possible values of T

Step 3: Choose the optimal value of T so that $\frac{d^2TVC_1(T)}{dT^2}>0$ and $\frac{d^2TVC_2(T)}{dT^2}>0$

Step 4: Using the optimal values of T in (10), (11) and (6) and find the total variable cost minimum and production time t_1.

7. NUMERICAL EXAMPLES

To illustrate the results let us apply the proposed method to solve the following numerical examples. The following parameters h = \$ 5/unit, C = \$5 / units, S = \$ 6 units, b = 1000, I_e = \$ 0.1 / \$ /year, I_p = \$ 0.15/\$/ year and Q = 0.3 are used in appropriate units. If A = \$ 50 / order, P = 5000 unit/unit time Then for T = 3.42, TVC_1 = \$ 1250.

8. CONCLUSION

This study presents a production inventory model for deteriorating items under permissible delay in payments with variable demand rate and small deterioration rate. In this work we found an optimal cycle time which give minimum total variable cost of the system corresponding to different value of parameters. This work done with linear demand rate. Numerical examples reveal that our optimization procedure is very accurate and rapid. From sensitive analysis we got sufficient variation in the value of parameters.

Briefly, results in this paper is better than the results of that works in which taken constant demand rate. Finally, a future study will incorporate more realistic assumption in the proposed model, such as variable deterioration rate, stochastic nature of demand and production rate which depends on both on-hand inventory and demand.

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