Fuzzy Goal Programming Techniques for Production Planning in Industry

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ABSTRACT

This paper presents the production planning problem in industry with different operational constraints, including strategic aim of the company, profit goal, limit on finishing and furnace hours needed, cups manufactured with target values being imprecise in nature. The fuzzy goal programming techniques is applied to maximize the production capacity, maximize the profit, minimize the extra finishing labor and furnace hours, and ensure the manufacturing capacity. The objective of this paper is to elaborate a plan which takes manager's preferences into account. The results illustrate the flexibility of the proposed model by adjusting goal priorities with respect to importance of each objective and the aspiration level with respect to desired target values. LINDO 14.0 optimizer solver is used to draw results of the problem.

Keywords: Goal Programming, Production Planning, Fuzzy Goal Programming, Satisficing.

1. INTRODUCTION

The production planning problems involves objectives as to either maximize profit or minimize cost and is formulated to a single-objective function in linear programming. But in real life there are multiple objectives involved with imprecise target values. In order to design an efficient production planning system, a good understanding of the environment in terms of customers, products and manufacturing processes is a must [16]. Production planning is a complicated task that requires cooperation among multiple functional units in any organization. Therefore, new tools for production planning are required that consider these issues. To achieve this, in this paper, a fuzzy goal programming technique is used to determine optimal production plans.

With fast computational growth [8], both linear and non-linear goal programming can be solved using well-developed software such as Linear Interactive and Discrete Optimization (LINDO, 2011) or meta-heuristics such as simulated annealing, genetic algorithms, tabu search and so on [10].

Goal programming (GP), was developed by Charnes and Cooper [1]. Lee [11] applied the goal programming approach to production planning and then to aggregate production planning. Ghosh et al. [4] presents a goal programming technique for nutrient management by determining the optimum fertilizer combination for rice production. Tamiz et al. [18] have studies the modeling approach of goal programming does not attempt to maximize or minimize the objective function directly as in the case of conventional linear programming. Instead of that the goal programming (GP) model seeks to minimize the deviations between the desired goals and the actual results to be obtained according to the assigned priorities. Dantzig [2, 3] developed linear programming under uncertainty and also provided solution of two-stage linear programs with uncertainty called as the stochastic programming. In stochastic programming, the parameters are random variables with known distribution. The use of fuzzy set theory in GP was first considered by Narasimhan [13, 14, 15], Hannan [5, 6, 7], Ignizio [9]. Rubin and Narsimhan [17] and Tiwari et al. [19, 20] have investigated various aspects of decision problem using FGP. An extensive review of these papers is given by Tiwari et al. in 1985.

2. FUZZY GOAL PROGRAMMING

The simple additive model in conventional GP for m goals

$G_i(x)$ with deviational variables d_i^+, d_i^- is defined as

$$\text{Minimize} \sum_{i=1}^{m} \left(d_i^+ + d_i^- \right)$$

$$d_i^+ \cdot d_i^- = 0,$$

$$d_i^+, d_i^-, x \geq 0, \ i = 1, 2, ..., m,$$

where g_i represents the aspiration level of the i-th goal.

2.1 Fuzzy Goal Programming Model

Now, further consider the FGP problem formulated as:

Find X

To satisfy $G_i(X) \gtrsim g_i, i = 1, 2, ..., m$, (2)

Subject to

$$AX \leq b$$

$$X \geq 0,$$

where X is an n-vector with components $x_1, x_2, ..., x_n$ and $AX \leq b$ are system constraints in vector notation. The symbol '$\gtrsim$' refers to the fuzzification of the aspiration level (i.e., approximately greater than or equal to). The i-th fuzzy goal $G_i(X) \gtrsim g_i$ in (2) signifies that the DM is satisfied even if less than the g_i upto certain tolerance limit is attained. A linear membership function μ_i for the i-th fuzzy goal $G_i(X) \gtrsim g_i$ can be expressed according to Zimmermann [21, 22] as

$$\mu_i = \begin{cases} 1 & \text{if } G_i(X) \geq g_i \\ \dfrac{G_i(X) - L_i}{g_i - L_i} & \text{if } L_i \leq G_i(X) \leq g_i \\ 0 & \text{if } G_i(X) \leq L_i \end{cases} \quad (3)$$

where L_i is the lower tolerance limit for the fuzzy goal $G_i(X)$. In case of the goal $G_i(X) \,\hat{}\, g_i$, the membership function is defined as

Subject to $G_i(x) + d_i^- - d_i^+ = g$

$$\mu_i = \begin{cases} 1 & \text{if } G_i(X) \leq g_i \\ \dfrac{U_i - G_i(X)}{U_i - g_i} & \text{if } g_i \leq G_i(X) \leq U_i \\ 0 & \text{if } G_i(X) \geq U_i \end{cases} \quad (4)$$

where U_i is the upper tolerance limit.

The additive model of the FGP problem (2) is formulated by adding the membership functions together as

$$\text{Maximize } V(\mu) = \sum_{i=1}^{m_1} \mu_i + \sum_{j=m_2}^{m} \mu_j$$

Subject to

$$\mu_i = \frac{G_i(X) - L_i}{g_i - L_i}$$

$$\mu_j = \frac{U_j - G_j(X)}{U_j - g_j}$$

$$AX \leq b,$$

$$\mu_i, \mu_j \leq 1; i = 1, 2, ..., m_1, j = m_1, ..., m,$$

$$X, \mu_i, \mu_j \geq 0, i = 1, 2, ..., m_1, j = m_1, ..., m,$$

(5)

where $V(\mu)$ is called the fuzzy achievement function or fuzzy decision function.

The weighted additive model is widely used in GP and multiobjective optimization techniques to reflect the relative importance of the goals/objectives. In this approach the DM assigns differential weights as coefficients of the individual terms in the simple additive fuzzy achievement function to reflect their relative importance, i.e., the objective function is formulated by multiplying each together. This leads to the following formulation, corresponding to (4):

$$\text{Maximize } V(\mu) = \sum_{i=1}^{m_1} w_i \mu_i + \sum_{j=m_2}^{m} w_j \mu_j$$

Subject to

$$\mu_i = \frac{G_i(X) - L_i}{g_i - L_i},$$

$$\mu_j = \frac{U_j - G_j(X)}{U_j - g_j}$$

$AX \leq b$,

$\mu_i, \mu_j \leq 1; i = 1, 2, ..., m_1, j = m_1, ..., m$,

$X, \mu_i, \mu_j \geq 0, i = 1, 2, ..., m_1, j = m_1, ..., m$,

$\qquad\qquad\qquad\qquad\qquad\qquad$ (6)

where w_i is the relative weight of the i-th fuzzy goal, w_j is the relative weight of the j-th fuzzy goal.

3. PRODUCTION PLANNING PROBLEM

Let us consider a problem proposed by Jones and Tamiz (2010) where a company produces three types of cups, termed grade A, grade B and grade C. Each grade A cup requires 2 hr of furnace time and 3 hr of finishing labor. Each grade B cup requires 3 hr of furnace time and 5 hr of finishing labor. Each grade C cup requires 4 hr of furnace time and 10 hr of finishing labor. A grade A cups yields a profit of £1.00, a grade B cup a profit of £1.50, and a grade C cup a profit of £2.20. The company currently has 1000 hr of furnace time and 2000 hr of finishing labor per day. They have a high level of demand and therefore they have a strategic aim of increasing production to 1000 cups per day by increasing the level of furnace and finishing hours available. The company wants to achieve the following goals:

Goal 1: Achieve the strategic aim of 1000 cups per day.

Goal 2: Achieve a profit of at least £1250.

Goal 3: Minimize the extra finishing and furnace hours needed.

Goal 4: Ensure that at least 300 of each type of cup is manufactured.

with the relative preference weights attached to all the goals as given below in Table 1:

Table 1: Preference weight

S. No.	Goal	Preference weight
1.	Achieve the strategic aim of 1000 cups per day	5.0
2.	Achieve a profit of at least £1250	3.0
3.	Minimize the extra finishing and furnace hours needed	2.0
4.	Ensure that at least 300 of each type of cup is manufactured	1.0

Assuming these goals to be fuzzy in nature, let the tolerance limit of four goals be (800, 1100, 2550, 5200, 280, 265, 260).

4. FORMULATION OF THE PROBLEM

Let

x_1 = Number of cups of grade A produced per day

x_2 = Number of cups of grade B produced per day

x_3 = Number of cups of grade C produced per day

Goals:

(i) Strategic aim of the company:

The company has the strategic aim of achieving 1000 cups per day which is assumed fuzzy in nature with the tolerance limit of 800 cups per day.

$$x_1 + x_2 + x_3 \gtrsim 1000 \qquad (7)$$

(ii) Profit goal:

The company desires to achieve a profit of at least £1250 with the tolerance limit of £1100.

$$x_1 + 1.5x_2 + 2.2x_3 \gtrsim 1250 \qquad (8)$$

(iii) Limit on finishing and furnace hours needed:

The company wants to limit the finishing and furnace hours to 1000 hours of furnace time and 2000 hours of finishing labor per day with the tolerance limit of 2550 and 5200, respectively.

$$2x_1 + 3x_2 + 4x_3 \lesssim 1000 \qquad (9)$$

$$3x_1 + 5x_2 + 10x_3 \lesssim 2000 \qquad (10)$$

(iv) Cups manufactured:

Company desires to manufacture at least 300 of each type of cup, again, assumed to be fuzzy in nature with the tolerance limit of (280, 265, 260).

$$x_1 \gtrsim 300 \qquad (11)$$

$$x_2 \gtrsim 300 \qquad (12)$$

$$x_3 \gtrsim 300 \qquad (13)$$

4.1. Fuzzy Goal Programming Problem is formulated as:

$$Maximize\ \frac{5}{11}\mu_1 + \frac{3}{11}\mu_2 + \frac{2}{11}(\mu_3 + \mu_4)$$

$$+\frac{1}{11}(\mu_5 + \mu_6 + \mu_7)$$

Subject to

$$\mu_1 = \frac{(x_1 + x_2 + x_3) - 800}{1000 - 800}$$

$$\mu_2 = \frac{(x_1 + 1.5x_2 + 2.2x_3) - 1100}{1250 - 1100}$$

$$\mu_3 = \frac{2550 - (2x_1 + 3x_2 + 4x_3)}{2550 - 1000}$$

$$\mu_4 = \frac{5200 - (3x_1 + 5x_2 + 10x_3)}{5200 - 2000}$$

$$\mu_5 = \frac{x_1 - 280}{300 - 280}$$

$$\mu_6 = \frac{x_2 - 265}{300 - 265}$$

$$\mu_7 = \frac{x_3 - 260}{300 - 260}$$

$x_i, \mu_j \geq 0\ (i = 1, 2, 3 ; j = 1, 2,...,7)$

$0 \leq \mu_j \leq 1$ (14)

4.2.   Computational Results

Using the data presented, the proposed weighted fuzzy goal programming model is tested using LINDO [12] software package and the results are shown in Table 2:

$$x_1 = 280.5, x_2 = 265, x_3 = 260$$

$$\mu_1 = 0.0275, \mu_2 = 1, \mu_3 = 0.099,$$

$$\mu_4 = 0.135, \mu_5 = 0.025, \mu_6 = 0, \mu_7 = 0.$$

Table 2: Interpretation of the Results

Goal	Description	Fuzzy Target Level	Satisfied	Achieved Value
1	Strategic aim	Tolerance limit of 800	Partial	805.5 ($x_1 + x_2 + x_3$)
2	Profit	Tolerance limit of 1100	Yes	1250
3	Minimize extra finishing and furnace hours	Tolerance limit of 2550	Partial	2396
		Tolerance limit of 5200	Partial	4766.5
4	No. of cups manufactured	A – Tolerance limit of 280	Partial	280.5
		B – Tolerance limit of 265	Partial	265
		C – Tolerance limit of 260	Partial	260

5. CONCLUSION

In this paper, we have solved the production planning problem through the weighted fuzzy goal programming (WFGP) technique representing the relative importance of each goal. Two major objectives with imprecise target values are optimized through fuzzy goal programming. The goals are relatively balanced by different goal programming variants. The similar technique could be used for short-term and intermediate production planning in other continuous process industries. The number of fuzzy goals can be increased based on the decision maker's desirability.

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