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Randić Index of Some Class of Trees with an Algorithm

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Abstract: The Randić index $R(G)$ of a graph G is defined as the sum of the weights $(d_G(u)d_G(v))^{-1/2}$ over all edges $e = uv$ of G. In this paper we have obtained the Randić index of some class of trees and of their complements. Also further developed an algorithmic technique to find Randić index of a graph.

Keywords: Algorithm, Degree of a vertex, Randić index, Tree.

1. INTRODUCTION

Let G be an undirected graph without loops and multiple edges with n vertices and m edges. Let $V(G) = \{v_1, v_2, \ldots, v_n\}$ be the vertex set of G and $E(G)= \{e_1, e_2, \ldots, e_n\}$ be the edge set of G. There are many types of indices, some based on distance of a graph and some other based on degrees of vertices of graphs. In 1975, the Randić index was proposed by the chemist Milan Randić [6] under the name ''branching index''. The Randić index $R(G)$ of a graph G is defined as the sum of the weights $(d_G(u)d_G(v))^{-1/2}$ over all edges $e = uv$ of G, where $d_G(u)$ is the degree of a vertex u in G. That is,

$$R(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}}$$

I. Gutman, et., al studied its mathematical properties and summarized in recent books [1,2]. The history of this index is described in [7, 8].It has been found that the Randić index correlates well with the harmonic index [4]. The expressions for the harmonic index and Randić index of the generalized transformation graphs and for their complement graphs were obtained in [5].The adjacency matrix of a graph G is the $n\times n$ matrix $A(G) = [a_{ij}]$, in which $a_{ij}= 1$ if v_i is adjacent to v_j and $a_{ij} = 0$, otherwise [3]. The harmonic index of some trees are obtained and an algorithm for the evaluation of the index is developed in [6].

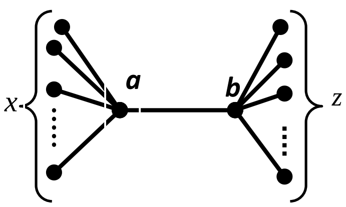


Fig.1: T_1

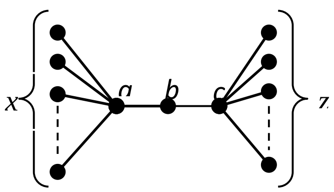


Fig.2: T_2

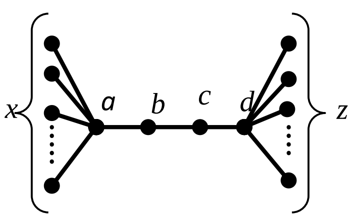


Fig3: T_3

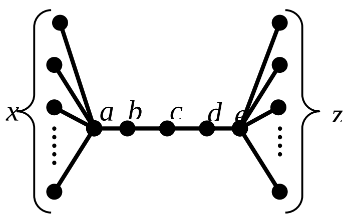


Fig.4: T_4

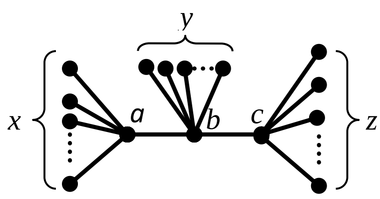


Fig.5: T_5

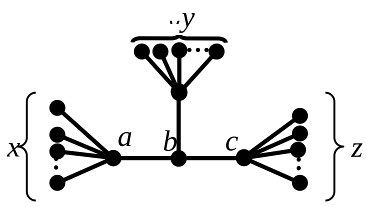


Fig.6: T_6

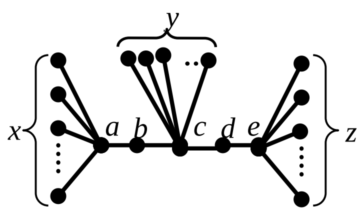


Fig.7: T_7

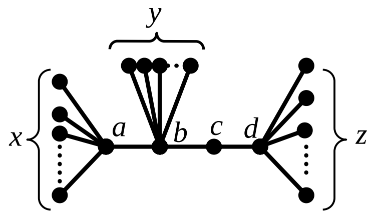


Fig.8: T_8

2. RESULTS:

Proposition 2.1: If T_1 is a tree with n vertices as shown in *Fig.1*, then the Randić index of T_1 is

$$R(T_1) = \frac{x}{\sqrt{x+1}} + \frac{n-x-2}{\sqrt{n-x-1}} + \frac{1}{\sqrt{(x+1)(n-x-1)}}.$$

Proof: Without loss of generality, consider the vertices a, b as shown in *Fig. 1*, where $d_{T_1}(a)=x+1$, $d_{T_1}(b)=n-x-1$. Partition the edge set $E(T_1)$ into 3 sets E_1, E_2 and E_3 such that $E_1=\{uv \mid d_{T_1}(u)=1 \text{ and } d_{T_1}(v)=x+1\}$, $E_2=\{uv \mid d_{T_1}(u)=1 \text{ and } d_{T_1}(v)=z+1\}$, $E_3=\{ab\}$. It is easy to see that $|E_1|=x$, $|E_2|=n-x-2$, $|E_3|=1$.

Therefore,

$$R(T_1) = \sum_{uv \in E(T_1)} \frac{1}{\sqrt{d_{T_1}(u)d_{T_1}(v)}}$$

$$= \sum_{uv\in E_1(T_1)} \frac{1}{\sqrt{d_{T_1}(u)d_{T_1}(v)}} + \sum_{uv\in E_2(T_1)} \frac{1}{\sqrt{d_{T_1}(u)d_{T_1}(v)}} + \sum_{uv\in E_3(T_1)} \frac{1}{\sqrt{d_{T_1}(u)d_{T_1}(v)}}$$

$$= \sum_{uv\in E_1(T_1)} \frac{1}{\sqrt{(x+1)(1)}} + \sum_{uv\in E_2(T_1)} \frac{1}{\sqrt{(n-x-2+1)(1)}} + \sum_{uv\in E_3(T_1)} \frac{1}{\sqrt{(x+1)(n-x-2+1)}}$$

$$= x\frac{1}{\sqrt{(x+1)}} + (n-x-2)\frac{1}{\sqrt{(n-x-2+1)}} + \frac{1}{\sqrt{(x+1)(n-x-2+1)}}$$

$$R(T_1) = \frac{x}{\sqrt{x+1}} + \frac{n-x-2}{\sqrt{n-x-1}} + \frac{1}{\sqrt{(x+1)(n-x-1)}}.$$

The following proposition 2.2 can be proved in analogous to the proposition 2.1.

Proposition 2.2: If T_i is a tree with n vertices as shown in *Fig. i*, then the Randić index of T_i,

$i=2,3,4$ as follows,

$$(i)\ R(T_2) = \frac{x}{\sqrt{x+1}} + \frac{n-x-3}{\sqrt{n-x-2}} + \frac{1}{\sqrt{2(x+1)}} + \frac{1}{\sqrt{2(n-x-2)}}$$

$$(ii)\ R(T_3) = \frac{1}{2} + \frac{x}{\sqrt{x+1}} + \frac{n-x-4}{\sqrt{n-x-3}} + \frac{1}{\sqrt{2(x+1)}} + \frac{1}{\sqrt{2(n-x-3)}}$$

$$(iii)\ R(T_4) = 1 + \frac{x}{\sqrt{x+1}} + \frac{n-x-5}{\sqrt{n-x-4}} + \frac{1}{\sqrt{2(x+1)}} + \frac{1}{\sqrt{2(n-x-4)}}$$

Proposition 2.3: If T_5 is a tree with n vertices as shown in *Fig.5*, then the Randić index of T_5 is

$$R(T_5) = \frac{x}{\sqrt{x+1}} + \frac{y}{\sqrt{y+2}} + \frac{n-x-y-3}{\sqrt{n-x-y-2}} + \frac{1}{\sqrt{(x+1)(y+2)}} + \frac{1}{\sqrt{(y+2)(n-x-y-2)}}.$$

Proof: Without loss of generality consider the vertices a,b,c as shown in *Fig. 5,* where $d_{T_5}(a)=x+1$, $d_{T_5}(b)=y+2$, $d_{T_5}(c)=z+1$. Partition $E(T_5)$into 5 sets $E_1, E_2, E_3, E_4,$ and E_5such that $E_1=\{uv|\ d_{T_5}(u)=1 \text{ and } d_{T_5}(v)=x+1\}$, $E_2=\{uv\ |\ d_{T_5}(u)=1 \text{ and } d_{T_5}(v)=y+2\}$, $E_3=\{uv|\ d_{T_5}(u)=1 \text{ and } d_{T_5}(v)=z+1\}$, $E_4=\{ab\}$, $E_5=\{bc\}$. It is easy to see that $|E_1|=x$, $|E_2|=y$, $|E_3|=z$, $|E_4|=|E_5|=1$.

Therefore,

$$R(T_5) = \sum_{uv\in E(T_5)} \frac{1}{\sqrt{d_G(u)d_G(v)}}$$

$$= \sum_{uv\in E_1(T_5)} \frac{1}{\sqrt{d_G(u)d_G(v)}} + \sum_{uv\in E_2(T_5)} \frac{1}{\sqrt{d_G(u)d_G(v)}} + \sum_{uv\in E_3(T_5)} \frac{1}{\sqrt{d_G(u)d_G(v)}}$$
$$+ \sum_{uv\in E_4(T_5)} \frac{1}{\sqrt{d_G(u)d_G(v)}} + \sum_{uv\in E_5(T_5)} \frac{1}{\sqrt{d_G(u)d_G(v)}}$$

$$= \sum_{uv\in E_1(T_5)} \frac{1}{\sqrt{(x+1)(1)}} + \sum_{uv\in E_2(T_5)} \frac{1}{\sqrt{(y+2)(1)}} + \sum_{uv\in E_3(T_5)} \frac{1}{\sqrt{(z+1)(1)}}$$
$$+ + \sum_{uv\in E_4(T_5)} \frac{1}{\sqrt{(x+1)(y+2)}} + \sum_{uv\in E_5(T_5)} \frac{1}{\sqrt{(y+2)(z+1)}}$$

$$= \frac{x}{\sqrt{x+1}} + \frac{y}{\sqrt{y+2}} + \frac{z}{\sqrt{z+1}} + \frac{1}{\sqrt{(x+1)(y+2)}} + \frac{1}{\sqrt{(y+2)(z+1)}}$$

Here we have $n=x+y+z+3$. By replacing $z=x+y+z+3$, the above equation reduces to

$$R(T_5) = \frac{x}{\sqrt{x+1}} + \frac{y}{\sqrt{y+2}} + \frac{n-x-y-3}{\sqrt{n-x-y-2}} + \frac{1}{\sqrt{(x+1)(y+2)}} + \frac{1}{\sqrt{(y+2)(n-x-y-2)}}.$$

Proposition 2.4: If T_i is a tree with n vertices as shown in *Fig. i*, then the Randić index of T_i, $i=6,7,8$ is as follows,

$$R(T_6) = \frac{x}{\sqrt{x+1}} + \frac{y}{\sqrt{y+1}} + \frac{n-x-y-4}{\sqrt{n-x-y-3}} + \frac{1}{\sqrt{3(x+1)}} + \frac{1}{\sqrt{3(y+1)}} + \frac{1}{\sqrt{3(n-x-y-3)}}$$

$$R(T_7) = \frac{x}{\sqrt{x+1}} + \frac{y}{\sqrt{y+2}} + \frac{n-x-y-5}{\sqrt{n-x-y-4}} + \frac{1}{\sqrt{2(x+1)}} + \frac{2}{\sqrt{2(y+2)}} + \frac{1}{\sqrt{2(n-x-y-4)}}$$

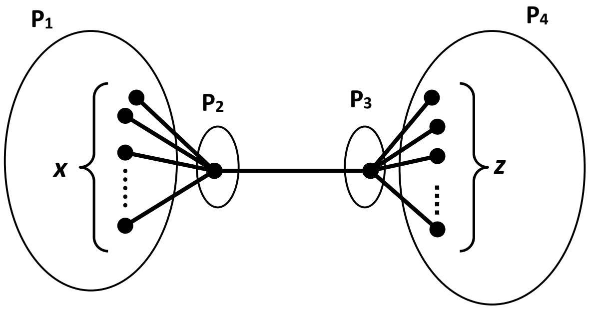
$$R(T_8) = \frac{x}{\sqrt{x+1}} + \frac{y}{\sqrt{y+2}} + \frac{n-x-y-4}{\sqrt{n-x-y-3}} + \frac{1}{\sqrt{(x+1)(y+2)}} + \frac{1}{\sqrt{2(y+2)}} + \frac{1}{\sqrt{2(n-x-y-3)}}$$

3. RESULTS FOR COMPLEMENTS

The complement of a graph G, denoted by $\bar{G}$ is a graph with vertex set $V(G)$ and two vertices in $\bar{G}$ are adjacent if and only if they are not adjacent in G [3].

Preposition 2.5: If $G=T_1$ is a tree with n vertices and m edges as shown in *Fig.1*, then the Randić index of complement of T_1 is

$$R(\bar{T}_1)=R(\bar{G})=\frac{n-3}{2}+\frac{n-x-2}{\sqrt{(n-2)(n-x-2)}}+\frac{x}{\sqrt{x(n-2)}}$$



Proof: Consider the partition $\mathbf{P_1}$, $\mathbf{P_2}$, $\mathbf{P_3}$ and $\mathbf{P_4}$ ofvertex set of T_1as shown in *Fig.9* . Easily we can note that $|P_1|=x$, $|P_2|=1$, $|P_3|=1$, $|P_4|=n-x-2$. Therefore,

$$R(\bar{T}_1)\quad=\sum_{uv\in E(\bar{T}_1)}\frac{1}{\sqrt{d_{\bar{G}}(u)d_{\bar{G}}(v)}}$$

$$=\sum_{uv\in(P_1)}\frac{1}{\sqrt{d_{\bar{G}}(u)d_{\bar{G}}(v)}}+\sum_{\substack{u\in(P_1)\\ v\in(P_3)}}\frac{1}{\sqrt{d_{\bar{G}}(u)d_{\bar{G}}(v)}}+$$
$$\sum_{\substack{u\in(P_1)\\ v\in(P_4)}}\frac{1}{\sqrt{d_{\bar{G}}(u)d_{\bar{G}}(v)}}+\sum_{\substack{u\in(P_2)\\ v\in(P_4)}}\frac{1}{\sqrt{d_{\bar{G}}(u)d_{\bar{G}}(v)}}+\sum_{uv\in(P_4)}\frac{1}{\sqrt{d_{\bar{G}}(u)d_{\bar{G}}(v)}}$$

$$=\sum_{uv\in(P_1)}\frac{1}{\sqrt{(n-1-1)(n-1-1)}}+\sum_{\substack{u\in(P_1)\\ v\in(P_3)}}\frac{1}{\sqrt{(n-1-1)(n-1-(n-x-1))}}$$
$$+\sum_{\substack{u\in(P_1)\\ v\in(P_4)}}\frac{1}{\sqrt{(n-1-1)(n-1-1)}}+\sum_{\substack{u\in(P_2)\\ v\in(P_4)}}\frac{1}{\sqrt{(n-1-(x+1))(n-1-1)}}$$
$$+\sum_{uv\in(P_4)}\frac{1}{\sqrt{(n-1-1)(n-1-1)}}$$

$$=\sum_{uv\in(P_1)}\frac{1}{(n-2)}+\sum_{\substack{u\in(P_1)\\ v\in(P_3)}}\frac{1}{\sqrt{(n-2)x}}+\sum_{\substack{u\in(P_1)\\ v\in(P_4)}}\frac{1}{(n-2)}$$
$$+\sum_{\substack{u\in(P_2)\\ v\in(P_4)}}\frac{1}{\sqrt{(n-x-2)(n-2)}}+\sum_{uv\in(P_4)}\frac{1}{(n-2)}$$

$$=\frac{n(n-1)}{2(n-2)}+\frac{x}{\sqrt{x(n-2)}}+\frac{x(n-x-2)}{(n-2)}+\frac{n-x-2}{\sqrt{(n-x-2)(n-2)}}$$
$$+\frac{(n-x-2)(n-x-3)}{(n-2)}$$

$$R(\bar{T}_1)=\frac{n-3}{2}+\frac{n-x-2}{\sqrt{(n-2)(n-x-2)}}+\frac{x}{\sqrt{x(n-2)}}$$

Proposition 2.6: If T_i is a tree with n vertices as shown in *Fig. i*, *i=2,3,4,5* then the Randić index of complement of T_i is as follows,

$$R(\bar{T}_2)=\frac{n^2-7n+12}{2(n-2)}+\frac{x}{\sqrt{(n-2)(n-3)}}+\frac{x}{\sqrt{(n-2)(x-3)}}+\frac{x}{\sqrt{(n-2)(x+1)}}$$
$$+\frac{1}{\sqrt{(n-x-2)(x+1)}}+\frac{n-x-3}{\sqrt{(n-x-2)(n-2)}}+\frac{n-x-3}{\sqrt{(n-3)(n-2)}}$$

$$R(\bar{T}_3)=\frac{n^2-9n+20}{2(n-2)}+\frac{2x}{\sqrt{(n-2)(n-3)}}+\frac{x}{\sqrt{(n-2)(x+2)}}$$
$$+\frac{1}{\sqrt{(n-x-2)(n-3)}}+\frac{1}{\sqrt{(n-x-2)(x+2)}}+\frac{n-x-4}{\sqrt{(n-x-2)(n-2)}}$$
$$+\frac{1}{\sqrt{(n-3)(x+2)}}+\frac{2(n-x-4)}{\sqrt{(n-3)(n-2)}}.$$

$$R(\bar{T}_4)=\frac{n^2-11n+30}{2(n-2)}+\frac{3x}{\sqrt{(n-2)(n-3)}}+\frac{x}{\sqrt{(n-2)(x+3)}}$$
$$+\frac{2}{\sqrt{(n-x-2)(n-3)}}+\frac{1}{\sqrt{(n-x-2)(x+3)}}+\frac{n-x-5}{\sqrt{(n-x-2)(n-2)}}$$
$$+\frac{1}{n-3}+\frac{2}{\sqrt{(n-3)(x+3)}}+\frac{3(n-x-5)}{\sqrt{(n-3)(n-2)}}.$$

$$R(\bar{T}_5)=\frac{x(x-1)}{2(n-2)}+\frac{x}{\sqrt{(n-2)(n-z-2)}}+\frac{x}{\sqrt{(n-2)(n-y-3)}}+\frac{xy}{n-2}$$
$$+\frac{xz}{n-2}+\frac{y}{\sqrt{(n-x-2)(n-2)}}+\frac{1}{\sqrt{(n-x-2)(n-z-2)}}+\frac{z}{\sqrt{(n-x-2)(n-2)}}$$
$$+\frac{y(y-1)}{2(n-2)}+\frac{y}{\sqrt{(n-z-2)(n-2)}}+\frac{yz}{n-2}+\frac{z}{\sqrt{(n-y-3)(n-2)}}+\frac{z(z-1)}{2(n-2)}$$

4. ALGORITHM:

➢ An algorithm to find the Randić index of a graph.

Step 1: START

Step 2: Declare: *a*[25][25],*d*[25],m as integers

sum1,*s*[25],sum, *ts=0* as floating points.

Step 3: Read *m*, *a*[*i*][*j*].

Step 4: Compute : Degree of each vertex of given graph

for *i* to *n*

d[*i*] ← *0*

for *j* to *n*

d[*i*] ← *d*[*i*] + *a*[*i*][*j*]

Display: Degree *d*[*i*] of vertex *i*

Step 5: Check the condition, if *a*[*i*][*j*]=*1* is true

Display: Vertex *i* is adjacent to vertex *j*

Step 6 : Multiply corresponding degrees of adjacent vertices

sum ← *d*[*i*]**d*[*j*]

Step 6: Display the sum of multiples of adjacent vertices degree

$ts \leftarrow ts+(1/\sqrt{sum})$

Step 7: Display the Randić index by dividing total sum *ts* by 2.

Step 8:STOP

Illustartion:

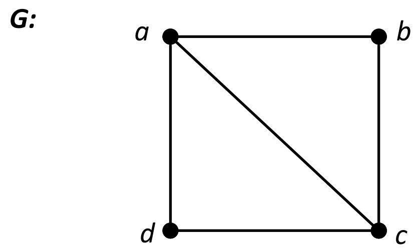


Fig. 10

We represent the graph G by adjacency matrix,

$$A(G) = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

In this matrix *a, b, c, d* represents the vertices of graph *G*. The element *1* in *A(G)* represents the adjacency between the vertices and *0* represents the non-adjacency between the vertices. Addition of elements of each row gives the degree of a corresponding vertex in *G*, ie., we get 3 by adding all the elements of a first row of adjacency matrix *A(G)*which is degree of vertex '*a'* in graph G. similarly we get other vertex degrees by adding corresponding row. Using this we calculate degree of each vertex and store it in *d[i]* by using for loop.

The outer loop iterates *i* times and the inner loop iterates *j* times, the statements inside the inner loop will be executed a total of *i*j* times. It is because, inner loop will iterate *j* times for each of the *i* iterations of the outer loop. This means the outer and inner loop are dependent on the problem size ie., here we considered size is *n*, the statement in the whole loop will be executed $O(n^2)$ times. In the loop int *i=0,* this will be executed only once. The time is actually calculated to *i=0* and not the declaration, *i<n* this will be executed *n+1* times, *i++* will be executed *n* times, *a[i][j]=1*, This will be executed *n* times (in worst case scenario).

As per the definition Randić index we multiply the degree of vertices which are adjacent, by adjacency matrix *A(G)*, we check the adjacency of one vertex to another by using if condition, then we multiply the degree of those adjacency vertices using *d[i]*,(This loop follows same procedure as explained for above loop so this also executed $O(n^2)$ times). Then we sum the multiplied valueof each adjacent vertices and each time we store the resulting value in one variable say *ts* as per the definition of Randić index,ie.,$1/\sqrt{sum}$. So for the above example we get final value of *ts* as 3.9266. We obtain the Randić index by dividing *ts* by 2.Therefore the Randić index of this example is 1.9633.

5. CONCLUSION

The results give explicit formulas for Randić index of certain class of trees and also for their complements. Further an algorithm with the help of adjacency matrix given to compute the Randić index of graph.

6. ACKNOWLEDGEMENTS
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